Are Currency Crises Low-State Equilibria?
An Empirical, Three-Interest-Rate Model

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The views expressed in this paper are those of the authors.  
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Abstract

Suppose that the dynamics of the macroeconomy were given by (partly) random fluctuations between two equilibria: “good” and “bad.” One would interpret currency crises (or recessions) as a shift from the good equilibrium to the bad. In this paper, the authors specify a dynamic investment-savings-aggregate-supply (IS-AS) model, determine its closed-form solution, and examine numerically its comparative statics. The authors estimate the model via maximum likelihood, using data for Argentina, Canada, and Turkey. Since the data show no support for the multiple-equilibrium explanation of fluctuations, the authors cast doubt on the third-generation models of currency crisis.

JEL classification: C62, E59, F41
Bank classification: Uncertainty and monetary policy

Résumé

Si l’on pose pour hypothèse que la dynamique macroéconomique est déterminée par des fluctuations partiellement aléatoires entre deux équilibres, le « bon » et le « mauvais », il est alors possible d’interpréter les crises de change (ou les récessions) comme le passage du bon au mauvais équilibre. Les auteurs formulent un modèle dynamique IS-AS (pour Investment-Savings et Aggregate Supply), trouvent sa solution analytique et examinent numériquement ses propriétés de statique comparative. Ils estiment le modèle par la méthode du maximum de vraisemblance au moyen de données relatives à l’Argentine, au Canada et à la Turquie. Comme leurs résultats ne confirment nullement que l’existence d’équilibres multiples est à l’origine des fluctuations, les auteurs émettent des doutes sur la validité des modèles de crise de change de troisième génération.

Classification JEL : C62, E59, F41
Classification de la Banque : Incertitude et politique monétaire
1. Introduction

One of the most controversial elements of Keynes’ General Theory (1936) has been the question of the “underemployment equilibrium.” Depending on one’s reading of passages in chapters 20 and 21, an economy subject to credit market breakdowns may suffer a temporary fall in output, or move to a second, underemployment equilibrium, from which active government intervention is required to assure growth and prosperity at the standard (full employment) equilibrium. This prompts the question as to whether there are one or two equilibria in the economy. A related question emerges from the third-generation literature on currency crisis: Can a currency crisis, interpreted as a breakdown of domestic credit markets following a devaluation, cause the economy to fall into an underemployment equilibrium in a manner consistent with Keynes’ description of many years ago? We propose a general class of investment-savings-aggregate-supply (IS-AS) models consistent with results from the literature on third-generation currency crisis and test them against data for Argentina and Turkey (middle-income nations that have faced currency crises in recent years) and for Canada (to guard against the possibility of excessive acceptance of the multiple-equilibrium hypothesis). We conclude that evidence does not exist to support any multiple-equilibrium hypothesis.

This paper develops a simple macroeconomic model of a small open economy subject to international shocks that may induce rapid depreciation in the nation’s currency and/or a drop in the nation’s income—in short, a currency crisis or a low-output state. This framework allows us to examine the interaction between particular forms of intermediate exchange rate regimes (“dirty floats”) and the potential use of that monetary policy to alter the likelihood of a currency crisis or a low-output state. We later discuss both the possible types of currency crises the nation could experience and how a monetary authority’s preference for exchange rate stability (as compared with other goals) alters its potential exposure to currency crises and low-output states.

One of the economic phenomena of the 1990s and 2000s has been the increased openness of the world economy, which has permitted nations to take full advantage of the opportunities offered by the free flow of commerce. The freer flow of financial assets has enabled most nations to adopt some form of freely floating or managed exchange rates that are fully convertible. But as these vast markets for assets and currencies have emerged, so has
the potential for destabilizing speculation and financial crises. In the last 10 years, major currency crises have plagued Mexico, Russia, East Asia, Turkey, and Argentina. These crises have led to large economic contractions, even on the scale of an economic depression. A nation experiencing such a crisis may reasonably expect to suffer a loss of 10–20 per cent of real gross domestic product (GDP) in one year. In comparison, the United States contracted by 45–50 per cent over four years between 1929 and 1933. It is difficult to understate the importance of a currency crisis. In the same vein, even if many developed nations do not experience currency crises per se, they may be exposed to prolonged periods of below-average growth that can, in some circumstances, be thought of as a low-output state. Our theoretical model provides for both possibilities in a unified framework.

The theory of exchange rate target zones dates back to Krugman (1991). His original target zone model predicts that the path of the current exchange rate will be smoother (exhibit lower volatility) than it would have been in a free float, because arbitrageurs alter their market behaviour in accordance with the predictable pattern of monetary intervention specified by an announced target zone policy. These target-zone models are called “S-curve” models, for the S-shaped graph of a targeted exchange rate against its hypothetical value if it had not been targeted. Bertola and Caballero (1992) offer a starkly different conclusion in the case of imperfect credibility (imperfect commitment to an announced policy). Exchange rates under a target zone may be more volatile if intervention is not fully credible; that is, if arbitrageurs cannot expect with certainty that the monetary authorities will follow the announced target zone policy. This case of increased volatility is described as an “inverted S-curve” model. This conclusion is modified if the monetary authority either increases the likelihood of defending the current regime or moderates the degree to which it shifts the new central parity upon realignment; Cornell (2003a) offers a detailed analysis. Target zone models are interesting in that they yield outcomes that differ little from freely floating exchange rates, except when exchange rates are near the edges of the band. Cornell (2003b) provides empirical evidence that target zones produce this effect. Target zone models are also theoretically useful, because they can be manipulated to describe a wide variety of managed exchange rate regimes.

A second relevant literature describes currency crises. A currency crisis is a situation in which unsustainable economic circumstances force a monetary authority to abandon a particular exchange rate regime (often a fixed or managed exchange rate regime) in favour of a freely floating exchange
rate. To analyze the crisis, theorists must specify the regime, identify the resources the monetary authority requires to maintain that regime, and identify circumstances when those resources are not available for defence of the regime.

Krugman (1979) provides the seminal work in this area with his model of how a currency crisis can be generated by excessive growth in aggregate demand over money demand (often due to excessively expansionary fiscal policy). These models have been termed “first-generation models of currency crisis.” The model is substantially refined into its canonical form by Flood and Garber (1984). The risk of a first-generation currency crisis appears to be unavoidable, due to the freedom that fiscal authorities have in setting tax rates and levels of government spending. Essentially, first-generation models describe the circumstances in which arbitrageurs profitably choose to attack a fixed exchange rate, and therefore when such attacks are likely to be successful; i.e., they generate a currency crisis.

Obstfeld (1996) develops an alternative model of currency crises, which has been dubbed a “second-generation” model. In Obstfeld’s paper, a speculative attack may be triggered by a perceived temptation of the monetary authority to devalue the currency, often because the monetary authority is known to follow a policy rule that gives weight to price and/or output stability, as well as exchange rate stability. The key concept is that if arbitrageurs perceive a possible devaluation in support of the other (assumed known) objectives of the monetary authorities, the crisis may occur. Therefore, second-generation models of currency crisis contain the possibility of self-fulfilling prophecies that a monetary authority may not be able to guard against with large stocks of foreign currency, as is possible in the first-generation case. A class of “third-generation” models, based primarily on financial sector crisis and moral-hazard issues, is also being developed, in the hopes that they contain elements that come closer to explaining the East Asian currency crisis of 1997–98. A brief review of that literature is available in chapter 8 of Sarno and Taylor (2002). As an example, Solomon (2003, 2004) proposes a third-generation model of currency crisis to explain the Turkish crisis of 2001.

We add to this literature by examining the interaction between typical monetary policy rules and the implied theoretical risk of a currency crisis. Most developed and many developing nations have a particular, well-known pattern to their monetary policy, even if they do not explicitly target the value of their currency. For example, New Zealand and Canada have prac-
ticed inflation targeting for at least a decade. (Inflation targeting can be modelled in a similar manner as exchange rate target zones.) On the other hand, the United States uses more flexibility in their decision-making process. The monetary policy of all these countries can be well approximated by variants of the rule first described by Taylor (1993). While the U.S. Federal Reserve places little importance on exchange rate stability, Lubik and Schorfheide (2003) provide evidence that the Bank of Canada included the exchange rate in its policy rule. In addition, Freedman (1994, 1995) describes the circumstances in which a monetary conditions index can be used as an operational target.

A second, and completely separate, motivation for this work is the improved conduct of monetary policy in open, industrialized nations. While many researchers are examining these problems from optimizing frameworks like dynamic stochastic general-equilibrium (DSGE) models, many policymakers operate using models in the IS-AS tradition. This paper uses an IS-AS approach in its attempt to appeal specifically to those monetary policymakers. Some monetary policy-makers believe that inflation targeting increases financial market instability, and that taking account of exchange rate movements when conducting monetary policy is important, but they are unable to provide a theoretical explanation for this phenomenon (Borio and White 2004). In addition, several leading economists, such as Howitt (2005), have long argued that coordination failures can serve as a possible propagation mechanism in Keynesian models, and that, in fact, such failures can be interpreted as being consistent with Keynes’s original idea of an underemployment equilibrium.

The model in this paper generates multiple equilibria. This is not a new phenomenon, especially in the study of notorious low-output periods, such as the Great Depression. This approach was first studied in principle by Jovanovic (1989) and in greater detail by Dagsvik and Jovanovic (1994). The latter conclude that the Great Depression did not constitute a second equilibrium. Not everyone agrees with this conclusion. For example, Cooper and Corbae (2002) model the Great Depression as a low-level equilibrium using standard DSGE techniques. On a more practical level, it is possible that the method of monetary policy can significantly alter how shocks are propagated, as Lubik and Schorfheide (2003, 2004) argue. In addition, Harrigan and Kuttner (2004) describe Japan’s macroeconomic stagnation in the 1990s as another possible example of a low-output state, and they justify the concerns of many central bankers in developed nations.
2. The Model

Let there be two nations, North and South. North is a large nation and uses the dollar as its national currency. We do not model the Northern economy in this paper; rather, we take its real interest rate ($r^W$) to be the world real interest rate. South is a small nation and it uses the peso as its national currency. The exchange rate quoted is $s$, which is the natural logarithm of the peso price of one dollar. We start by constructing the foundations of such a model, allowing for later modification of elements of the model as suggested by empirical evidence.

2.1 Credit (loanable funds) market

Investment is $I(t) = \alpha_1 - \beta_1 r^C(t)$. We treat investment prospects in South as exogenous and address how the capital markets affect access to those prospects. Let $r^C(t)$ be the prevailing real interest rate in South’s credit market.

The sum of national savings and net capital outflows (i.e., capital account deficit) is $S(t) = \alpha_2 + \beta_2 r^C(t) + \lambda y(t) + \theta y(t-1) + \kappa s(t) - \phi[s(t) - s(t-1)]^2 + \epsilon(t)$. The sum of national savings and net capital outflows depends positively on past (log) income (because, in developing economies, people may be credit-constrained and therefore consumption and income will covary strongly), positively on the exchange rate (most strongly in the case of net capital outflows), negatively on exchange rate volatility (uncertainty leads to fears of bank balance sheets deteriorating, leading in turn to credit tightening), and positively on an IS shock that we will interpret as a public savings shock, rather than “animal spirits.”

Balance-of-payments accounting tells us that $I - S = KA$, or $I = S + KA$. In other words, the credit market is in continuous equilibrium; foreign positions in South emerge out of an imbalance between Southern investment opportunities and the stock of Southern national savings. We assemble this equilibrium by substitution, noting that $\alpha = (\alpha_1 - \alpha_2)$ and $\beta = (\beta_1 + \beta_2)$:

$$\alpha_1 - \beta_1 r^C(t) = \alpha_2 + \beta_2 r^C(t) + \lambda y(t) + \theta y(t-1) + \kappa s(t) - \phi[s(t) - s(t-1)]^2 + \epsilon(t),$$

or

$$\alpha - \beta r^C(t) - \lambda y(t) - \theta y(t-1) - \kappa s(t) + \phi[s(t) - s(t-1)]^2 - \epsilon_y(t) = 0. \ (1)$$
Generalizing, we are interested in preserving the core elements of equation (1), but in changing particular terms, as the data for each nation make clear. The multiple equilibrium result depends critically on the presence of a term employing the square of the exchange rate, but that term can be in this or, alternatively, in another of the equations. We can also imagine adding the credit interest rate in this term, and possibly interaction terms such as $y(t-1)s(t)$ to capture nation-specific phenomena. Some of those alternatives are explored below in the empirical analysis.

2.2 Inflation dynamics equation

Following the new neoclassical synthesis (NNS) literature (as typified by such works as Woodford 2003), we posit a dynamic model of inflation determination. Modifying the model of Ball (1999) slightly, we have

$$\pi(t) = \rho \pi(t-1) + \gamma [y(t-1) - y_P(t)] + \delta [s(t) - s(t-1)] + \epsilon_\pi(t),$$

or

$$-\pi(t) + \rho \pi(t-1) + \gamma [y(t-1) - y_P(t)] + \delta [s(t) - s(t-1)] + \epsilon_\pi(t) = 0. \quad (2)$$

In this model, current inflation is determined by the sum of past inflation, the prior period’s output gap, the current depreciation (depreciations are inflationary for all the usual reasons), and a random-shock term reflecting sticky prices or monetary misperceptions. The coefficient $\rho$ could be between zero and 1, which means that, under good policy, inflation tends to zero, or $\rho$ could be exactly 1, which means that inflation is a random walk with a (policy-affectable) drift.

Note that even if output stays exactly at its long-run potential, inflation can still occur in an open economy if and only if pressure is exerted via the exchange rate channel.

Alternative renditions of this equation can include the exchange rate (particularly for small open economies), the square of the exchange rate (to generate multiple equilibria), and interaction terms. Further, one may choose to drop the $y(t-1) - y_P(t)$ term in favour of other methods of accounting for persistent growth in real GDP.
2.3 Interest rate effect on exchange rates

Along the lines suggested by Ball (1999), we posit that

\[ s(t) = \chi[r^C(t) - r^M(t)] + s(t-1) + \epsilon_s(t), \]

or

\[ -s(t) + \chi[r^C(t) - r^M(t)] + s(t-1) + \epsilon_s(t) = 0. \]  

2.4 Monetary policy

Each period, South’s central bank sets a short-term (policy) interest rate \( r^M(t) \), which somewhere in the background involves manipulation of the monetary base and/or the money multiplier. Later in this paper, we will attempt to model how a monetary authority might choose to act. In this section, we show how the long-term (credit) interest rate interacts with financial markets in equilibrium. Specifically, we characterize the evolution of the credit rate as follows:

\[ r^C(t) = r^C(t-1) + r_0 + r_1\pi(t) + r_2[y(t) - y(t-1)]. \]

We argue that the long-run (credit) interest rate is (approximately) a random walk with a constant drift, \( r_0 \), which may be shocked by either inflation or economic growth. This becomes

\[ -r^C(t) + r^C(t-1) + r_0 + r_1\pi(t) + r_2[y(t) - y(t-1)] = 0. \]
2.5 Short- and long-run equilibrium

Short-run equilibrium is given by $r^M = r^C$, so that monetary policy directly impacts credit market activity. Running this through the IS equation, $y(t)$ is determined by $r^M(t)$, which determines $r^C(t)$.

In the long run, $r^M$ is irrelevant. The nominal interest rate that will prevail is $r^C$, which in the long run is given by $r^W + Z(t)$, where $Z$ is the risk premium on assets in South (relative to North). Let $Z(t) = r^C(t) - r^W$. We interpret $Z(t)$ as a risk premium, or a residual after policy decisions have been made. We can define the output gap by this gap, and let it enter our equation describing inflation dynamics.

We would like to characterize potential output ($y_P$) as growing at a steady rate of growth, which in logs is given by

$$y_P(t) - y(0) - \zeta t = 0.$$  \hspace{1cm} (5)

Potential output equals initial-period output plus a constant growth rate of $100 \exp(\zeta)$ per cent in levels (which is $\zeta$ in logs).

In alternative empirical models, we drop references to equation (5), but the model retains the same basic flavour and class of results.

2.6 Derivation of multiple equilibria

The natural question that arises is whether this theoretical model can generate multiple equilibria. Equations (1) – (5) form a system that describes the relationships among key economic variables in the economy. The equations are (in order):

\begin{align*}
\alpha - \beta r^C(t) - \lambda y(t) - \theta y(t - 1) - \kappa s(t) + \phi [s(t) - s(t - 1)]^2 - \varepsilon_y(t) &= 0 \\
-\pi(t) + \rho \pi(t - 1) + \gamma [y(t - 1) - y_P(t)] + \delta [s(t) - s(t - 1)] + \varepsilon_\pi(t) &= 0 \\
-s(t) + \chi [r^C(t) - r^M(t)] + s(t - 1) + \varepsilon_s(t) &= 0 \\
-r^C(t) + r^C(t - 1) + r_0 + r_1 \pi(t) + r_2 [y(t) - y(t - 1)] &= 0 \\
y_P(t) - y(0) - \zeta t &= 0.
\end{align*}

By repeated substitutions, the three principal variables ($s$, $y$, and $\pi$) can be written as functions of their own lags, lagged other variables, contemporaneous policy choices, and contemporaneous shocks. While solving this
system can be quite complicated, one can intuitively visualize the solution procedure.

First, the multiple-equilibrium nature of the solution can be considered by studying equations (1), (3), and (4). Substituting (4) into (3) suggests that the solution for \( s(t) \) is a function of \( y(t) \). Substituting this combination into equation (1) squares the solution for \( s(t) \), including \( y(t) \), thus making the final form of equation (1) a function of \( y^2(t) \) and \( y(t) \). This implies that we will be left with an equation of the general form,

\[
a_2y^2(t) + a_1y(t) + a_0 = 0, \tag{6}
\]

where the values of \( a_0, a_1, \) and \( a_2 \) are complicated, real-valued functions of the parameters of the model, the shock terms, and the exogenous variables (including prior-period variable values). The solutions for \( a_0, a_1, \) and \( a_2 \), while complicated, are not functions of the remaining endogenous variables, so that the solutions to \( s(t), \pi(t), \) and \( y_P(t) \) can be written as functions of the parameters, the shock terms, the exogenous variables (including prior values), and a solution to \( y(t) \). Those solutions are as follows:

\[
\begin{align*}
 r^C(t) & = [-1/ (\delta \beta r_1 + \delta \kappa \chi r_1 - \phi)] \times \\
 & \quad [\phi \gamma(t - 1) - r_1 \phi \gamma(t - 1) - r_1 \delta \kappa \chi r^M(t) + r_1 \lambda \delta y(t) + r_1 \phi \rho \pi(t - 1) \\
 & + r_1 \delta \theta y(t - 1) + r_1 \delta \kappa \xi(t) + r_1 \delta \kappa s(t - 1) - r_1 \phi \gamma(0) + r_1 \phi \gamma y(t - 1) \\
 & + \phi r^C(t - 1) + r_1 \phi \varepsilon(\pi(t) + r_2 \phi y(t) - r_1 \delta \varepsilon_y(t) - r_1 \delta \alpha); \\
 s(t) & = [-1/ (\delta \beta r_1 + \delta \kappa \chi r_1 - \phi)] \times \\
 & \quad [\phi \varepsilon_s(t) - \chi r_1 \phi \gamma(0) + \chi r_1 \phi \gamma y(t - 1) - \delta \beta r_1 \varepsilon_s(t) + \chi \phi r_2 y(t) \\
 & + \chi r_1 \delta g y y(t - 1) + \chi r_1 \phi \rho \pi(t - 1) - \chi r_1 \delta \alpha + \chi r_1 \phi \varepsilon(\pi(t) \\
 & + \chi r_1 \delta \lambda y(t) - \chi r_1 \phi \gamma \xi(t) - \chi r_1 \delta \varepsilon_y(t) + \chi \beta \delta r_1 r^M(t) - \delta \beta r_1 s(t - 1) \\
 & + \phi s(t - 1) - \phi \chi r_2 y(t - 1) - \phi \chi r^M(t) + \phi \chi r_0 + \phi \chi r^C(t - 1));
\end{align*}
\]
\[ \pi(t) = \left[-1/(\delta\beta r_1 + \delta\kappa r_1 - \phi)\right] \times \\
\quad \left[-\delta\kappa\chi r_2 y(t) + \beta\delta r_2 y(t) + \lambda\delta y(t) - \phi\gamma t \phi \rho \pi(t-1) + \phi \varepsilon z(t) \right] \\
\quad + \phi\gamma y(t-1) - \delta\kappa\chi r_2 y(t-1) - \delta\kappa\chi r^M(t) + \delta\kappa\chi r_0 - \delta\beta r_2 y(t-1) \\
\quad + \delta\kappa\chi r^C(t-1) + \delta\beta r_0 + \delta\theta y(t-1) + \delta\kappa\varepsilon z(t-1) + \delta\kappa s(t-1) \\
\quad + \delta\beta r^C(t-1) - \phi\gamma y(0) - \delta\alpha - \delta\varepsilon y(t); \]

\[ y_P(t) = y(0) + \zeta t. \]

Note that the values of these parameters are constrained by the estimated gap between the full-employment and underemployment equilibria. In principle, we would like to identify a full-employment equilibrium, \( y = y_P \), and an underemployment equilibrium, \( y = \nu y_P \), where \( \nu \) denotes the size of the gap between the two equilibria. For example, an underemployment equilibrium in the American-Canadian data that we study might be thought of as a level of GDP of at least 5 per cent below potential output.\(^1\) In that case, \( Y_P \) would give us full-employment output in levels and \( 0.95Y_P(\nu = 0.95) \) would reflect a level of GDP that is 5 per cent below potential output, so that, in logs, \( y_P \) gives us full employment output and \( y_P + \ln(0.95) \) gives us the underemployment level of output. Thus, equation (6) must provide the same solution as \( \left[y - y_P\right]\left[y - (y_P - \nu y_P)\right] \) or simply \( \left[y - y_P\right]\left[y - y_P(1 - \nu)\right] \). Setting that expression equal to zero and solving, we find that \( y^2 - (\nu y_P)y + ((1 - \nu)y_P^2) \). Dividing equation (6) through by \( a_2 \), we find that our coefficients \( a_2, a_1, \) and \( a_0 \) have the following restrictions:

\[ \frac{a_1}{a_2} = -\nu y_P \quad \text{and} \quad \frac{a_0}{a_2} = (1 - \nu) y_P^2. \]

In turn, these restrictions may be combined to form the following more general restriction:

\[ \frac{a_0a_2}{a_1} = \frac{(1 - \nu)}{\nu^2}. \]

In our case, where \( \nu = 0.95 \), this implies that \( (a_0a_2)/a_1 \approx 0.0554 \).

\(^1\) We interpret Canada as South and the United States as North, which is geographically correct in some parts of the Midwest!
3. Selection of Monetary Policy

In this section, we begin to model equilibrium selection and robust policy; that is, policy that is sensible in a wide variety of possible macroeconomic scenarios. The policy rule is the result of minimizing a loss function subject to the equations of the model. Let this loss function be described as follows:

\[ L = E_{t-1} \left[ \rho_y [y(t) - y]_t^2 + \rho_y [y(t) - y_F(t)]_t^2 + \rho_y [r^M(t) - r^M(t - 1)]_t^2 + \rho_y [s(t) - s(t - 1)]_t^2 \right]. \]  (7)

Since there are multiple equilibria in this model, we define a crisis equilibrium as the low-output equilibrium; that is, the underemployment equilibrium. Let \( C(t) \) denote the crisis indicator variable, where a value of 1 denotes the presence of a crisis (a low-output state) and zero denotes the absence of a crisis (a high-output state). The crisis should have some exogeneity, some endogeneity, and some persistence. Let the probability of a crisis state be a Gaussian-valued random variable, the cutoff value of which is given by

\[ Pr[C(t) = 1] = \Phi(c_0 + c_1 (r^M - r^C)(t - 1) + c_2 F(t) + c_3 C(t - 1)). \]  (8)

In this formulation, the probability of a crisis depends upon the constant \( c_0 \), the last period’s gap between the credit and policy interest rates, current foreign investment, and whether there was a crisis last period. The question of the proper interpretation of \( c_0 \) remains, which we consider to indicate the overall foreign investment climate, or at least the portion of the probability of a crisis left unexplained by the other three factors that we model.

Let the law of motion for foreign investment \((F(t))\) be given by

\[ F(t) = f_0 + f_1 (r^C - r^W)(t - 1) + f_2 E_{t-1}[s(t) - s(t - 1)] + f_3 C(t). \]  (9)

In this equation, foreign investment is a function of a constant \( f_0 \), South’s country-specific risk premium, the last period’s expected depreciation, and whether there was a crisis during the last period. The constant \( f_0 \) has a clearer interpretation here, and it suggests a desirable level of foreign investment in “calm times” (when the risk premium is zero, there is no expected depreciation, and there is no crisis).

Our concept of monetary policy in this paper is guided by this vision. The conduct of modern monetary policy is frequently characterized by linear policy rules that give different weight to various policy objectives, such as
achieving an inflation target, closing the output gap, and possibly smoothing changes in interest rates or exchange rates. One may derive such a policy rule by minimizing a “loss function,” a function that assigns penalties for failing to reach one of these targets, subject to fixed (linear) relationships among the real and nominal variables of the economy, such as an IS curve, an aggregate supply curve, or a Phillips curve. As Ball (1999) shows, each of these fixed relationships can be suitably modified to reflect aspects of an open economy.

Several assumptions underly the loss-function framework. First, monetary policy causes real and nominal variables to react in predictable ways. Taylor (1993) shows that some simple rules can lead to indeterminacy; his suggestion is to avoid this class of rules. But what if there was fundamental indeterminacy in the real economy, independent of the chosen monetary policy rule? For example, it is possible for the real side of the economy to be characterized by two equilibria: “good” and “bad.” The non-uniqueness of the equilibrium comes from the short-run and long-run aspects of the credit market. The short-run credit market deals primarily with credit for the purchase of short-term consumption goods. The long-run credit market deals primarily with credit for the purchase of long-term capital by firms. Market clearing in these two markets leads necessarily to multiplicity. Second, even in models where the real economy is subject to multiple equilibria, equilibrium selection is typically modelled as purely random, or as something that can be affected by events and policy choices. But the reality is more complex. A bad equilibrium can be the result of what must be viewed from the perspective of the open economy as bad luck – speculators large in relation to the market decide to make a one-way bet against the current level of the nominal exchange rate, causing a depreciation, inflation to rise, and output to fall. This decision may be precipitated, or exacerbated, by the decisions of firms with large foreign investments in the open economy. But monetary policy can also play a role in causing or preventing the bad equilibrium. Although these bad equilibria are relatively rare, they are fairly persistent and tend to impose large costs on the economy. Should a policy-maker ignore these considerations in setting monetary policy?
4. Econometric Analysis

The greatest econometric problem facing this work is the analysis of a multiple-equilibrium model. Traditional econometrics is ill-equipped to handle such challenges. In particular, one would ordinarily estimate the outcomes of such equilibria separately, so that if a crisis occurred with 5 per cent probability and one had 200 data points (for example, quarterly data for a 50-year span), then one would effectively have to base parameter estimates on 10 data points, which hardly seems sufficient for even a single-variable model, let alone the model described above.

4.1 The data

The initial estimation of the model is conducted using U.S.-Canadian data, for three reasons. First, estimating this model for a developed nation guards against the possibility of a framework that excessively accepts multiple equilibria, so that the issue is focused on true currency crises. Second, it is possible that the underemployment equilibrium may differ in severity between developed and developing nations. For example, a developed nation suffering a crisis may experience a 5 per cent fall in GDP, while a developing nation may experience a 20 per cent fall in GDP. Third, this estimation is a convenient way to confirm the qualitative results found in the other two estimations.

For consistency of data and policy regimes, we study quarterly data from 1982Q3 through 2004Q2, which yields 88 observations. Accounting for the need to establish an initial period (assumed to be 1982Q3), we effectively have 87 data points for econometric analysis. We are prevented from studying higher-frequency macro data by the limits of observation on national income and product accounting (NIPA) and balance-of-payments (BOP) data, which of course are needed to study real GDP and foreign investment flows.

Our macroeconomic data are, unless otherwise noted, drawn from the CANSIM database operated by Statistics Canada, and downloaded by Bank of Canada staff. CANSIM does not report American-Canadian bilateral BOP data, so the analogous figures from the U.S. Bureau of Economic Analysis are substituted. The long-term world credit interest rate was drawn by using the U.S. ten-year Treasury bill rate, which is available from the Federal Reserve Board of Governors.

Our second estimation of the model is conducted using U.S.-Turkish
data, including the period of the crisis of 2001. The data are publicly
available on the website of the Central Bank of the Republic of Turkey, at
<http://www.tcmb.gov.tr/>. Our quarterly data set begins in 1988Q1 and
ends in 2004Q4, which yields 68 observations.

Our third estimation of the model is conducted using U.S.-Argentine
data, including the period of the crisis of 2001-02. The data are pub-
licly available at two locations: first, on the website of the Central Bank
of the Republic of Argentina, at <http://www.bcra.gov.ar/>, and second,
on the website of the Argentine Ministry of Economy and Production, at
and ends in 2004Q4, which yields 48 observations.

4.2 Calibration and simulation

Our first approach involves calibration and simulation. Although this is not
meant to be an estimation technique per se, it does attempt to ensure that
the model offers qualitative solutions of the type desired to estimate the
data in question. We begin by offering common estimates of many of the
variables encoded in the model, then simulating the variables by generating
simulated draws of the shocks specified, and finally printing out and even
graphing the simulated draws of the model to check for internal consistency
and the overall qualitative nature of the results. We learn two lessons from
this exercise: (i) qualitatively, the model behaves much as predicted, and (ii)
the model is quite sensitive to the parameter values chosen. For example,
we quickly generate a model that operates in a full-employment level (plus
or minus minor shocks) for long periods of time that suffers rapid crises
followed by rapid recovery. As currently constituted, the model does not
have sufficient dynamics to describe a prolonged crisis. This may be induced
by either introducing autoregressive shocks or artificially boosting the value
of \( c_3 \) in equation (8).

4.3 Full-information maximum likelihood

Our second approach involves system-wide regression of equations (1) – (5).
Least squares breaks down quickly in this environment, and it is clear that
the endogeneity problem would plague our results greatly. A promising tech-
nique with which to clarify the issue is full-information maximum likelihood
Rather than run a fixed model once for each nation, we find it useful to modify the model several times for each nation to fit the data more closely, and run it again through FIML to verify the improved econometric fit. Using this technique, we realize seven sets of results: three for Canada and two each for Argentina and Turkey. The various specifications consider issues such as whether there is an independent, deterministic trend in inflation (important for these countries, since inflation has been declining over the sample), and whether the level or the volatility of the exchange rate is more relevant as a determinant of aggregate output. In all seven cases, the multiple-equilibrium condition holds. The solution given in section 2.6 nests the seven cases estimated.

4.3.1 Results for Canada

Canada, Estimation 1

\[ y(t) = c_{13} y(t - 1) + c_{14} s^2(t) \]
\[ \pi(t) = c_{22} \pi(t - 1) + c_{23} y(t - 1) + c_{24} s^2(t) \]
\[ s(t) = c_{32} s(t - 1) + c_{33} r^C(t) \]
\[ r^C(t) = c_{42} r^C(t - 1) + c_{43} s^2(t) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Point est.</th>
<th>Std. error</th>
<th>z-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(13)</td>
<td>0.842239</td>
<td>0.061950</td>
<td>13.59555</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(14)</td>
<td>-0.397921</td>
<td>0.471733</td>
<td>-0.843530</td>
<td>0.3989</td>
</tr>
<tr>
<td>C(22)</td>
<td>-0.081257</td>
<td>0.112783</td>
<td>-0.720467</td>
<td>0.4712</td>
</tr>
<tr>
<td>C(23)</td>
<td>0.028174</td>
<td>0.021935</td>
<td>1.284453</td>
<td>0.1990</td>
</tr>
<tr>
<td>C(24)</td>
<td>-0.095220</td>
<td>0.125955</td>
<td>-0.755987</td>
<td>0.4497</td>
</tr>
<tr>
<td>C(32)</td>
<td>0.868562</td>
<td>0.061503</td>
<td>14.12230</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(33)</td>
<td>-0.399660</td>
<td>0.424031</td>
<td>-0.942526</td>
<td>0.3459</td>
</tr>
<tr>
<td>C(42)</td>
<td>0.746885</td>
<td>0.077262</td>
<td>9.666877</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(43)</td>
<td>-0.184669</td>
<td>0.365429</td>
<td>-0.505349</td>
<td>0.6133</td>
</tr>
</tbody>
</table>

The log likelihood for the system is 1322.259 and the determinant residual covariance for the system is \(7.40 \times 10^{-19}\).

\(^2\)All FIML estimations are performed in Eviews version 5.1.
Canada, Estimation 2

\[ y(t) = c_{11} + c_{12}r^C(t) + c_{13}y(t - 1) + c_{14}s(t) \]

\[ \pi(t) = c_{21}\pi(t - 1) + c_{22}(y(t - 1) - y^P) + c_{23}t \]

\[ s(t) = s(t - 1) + c_{31}\left[r^C(t) - r^M(t)\right] \]

\[ r^C(t) = r^C(t - 1) + c_{42}s^2(t) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Point est.</th>
<th>Std. error</th>
<th>z-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(11)</td>
<td>0.338610</td>
<td>0.303799</td>
<td>1.114586</td>
<td>0.2650</td>
</tr>
<tr>
<td>C(12)</td>
<td>-0.000352</td>
<td>0.092746</td>
<td>-0.003793</td>
<td>0.9970</td>
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<tr>
<td>C(13)</td>
<td>0.987278</td>
<td>0.010693</td>
<td>92.32539</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(14)</td>
<td>0.089250</td>
<td>0.073525</td>
<td>1.213874</td>
<td>0.2248</td>
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<tr>
<td>C(21)</td>
<td>0.865834</td>
<td>0.032347</td>
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<tr>
<td>C(22)</td>
<td>0.070829</td>
<td>0.037854</td>
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<td>C(23)</td>
<td>-0.000509</td>
<td>0.000276</td>
<td>-1.843243</td>
<td>0.0653</td>
</tr>
<tr>
<td>C(31)</td>
<td>25.49189</td>
<td>67.64107</td>
<td>0.376870</td>
<td>0.7063</td>
</tr>
<tr>
<td>C(42)</td>
<td>3654.406</td>
<td>12076664</td>
<td>0.000303</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

The log likelihood for the system is 903.8338 and the determinant residual covariance for the system is \(2.98 \times 10^{-5}\).

Canada, Estimation 3

\[ y(t) = c_{11} + c_{13}y(t - 1) + c_{14}s(t) \]

\[ \pi(t) = c_{21}\pi(t - 1) + c_{22}(y(t - 1) - y^P) + c_{23}t \]

\[ s(t) = s(t - 1) + c_{31}\left[r^C(t) - r^M(t)\right] \]

\[ r^C(t) = r^C(t - 1) + c_{42}s^2(t) \]
The determinant residual covariance for the system is $1.86 \times 10^{-17}$.

### 4.3.2 Results for Argentina

Argentina, Estimation 1

\[
y(t) = c_{11} + y(t-1)s(t)
\]

\[
\pi(t) = c_{22} + y(t-1)s(t) + c_{24}s(t)
\]

\[
s(t) = s(t-1) + c_{33}r^M(t-1)
\]

\[
r^C(t) = c_{41} + c_{42}r^C(t-1)s(t) + c_{44}r^C(t-1)
\]

The log likelihood for the system is -161.2804 and the determinant residual covariance for the system is 0.013045.
Argentina, Estimation 2

\[ y(t) = c_{11} + y(t - 1) \]

\[ \pi(t) = c_{22} + y(t - 1)s(t) + c_{24}s(t) \]

\[ s(t) = s(t - 1) + c_{32}r^C(t) + c_{33}r^M(t - 1) \]

\[ r^C(t) = r^C(t - 1) + c_{45}s^2(t) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Point est.</th>
<th>Std. error</th>
<th>z-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(11)</td>
<td>0.005962</td>
<td>0.011895</td>
<td>0.501257</td>
<td>0.6162</td>
</tr>
<tr>
<td>C(22)</td>
<td>-276.7790</td>
<td>47.03432</td>
<td>-5.884619</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(24)</td>
<td>9196.497</td>
<td>1559.404</td>
<td>5.897443</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(33)</td>
<td>0.005466</td>
<td>0.001770</td>
<td>3.087473</td>
<td>0.0020</td>
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<tr>
<td>C(32)</td>
<td>-0.002673</td>
<td>0.002421</td>
<td>-1.104050</td>
<td>0.2696</td>
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<tr>
<td>C(45)</td>
<td>-0.378281</td>
<td>1.184839</td>
<td>-0.319268</td>
<td>0.7495</td>
</tr>
</tbody>
</table>

The log likelihood for the system is -162.1074 and the determinant residual covariance for the system is 0.013507.

4.3.3 Results for Turkey

Turkey, Estimation 1

\[ y(t) = y(t - 1) + c_{14}s(t) \]

\[ \pi(t) = c_{21}\pi(t - 1) + c_{22}(y(t - 1) - y^P) + c_{23}t \]

\[ s(t) = s(t - 1) + c_{31}r^C(t) \]

\[ r^C(t) = r^C(t - 1) + c_{42}s^2(t) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Point est.</th>
<th>Std. error</th>
<th>z-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(14)</td>
<td>81.18124</td>
<td>23.12796</td>
<td>3.510090</td>
<td>0.0004</td>
</tr>
<tr>
<td>C(21)</td>
<td>0.627456</td>
<td>0.129442</td>
<td>4.847383</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(22)</td>
<td>-1.89 \times 10^{-5}</td>
<td>8.74 \times 10^{-6}</td>
<td>-2.166668</td>
<td>0.0303</td>
</tr>
<tr>
<td>C(23)</td>
<td>0.018732</td>
<td>0.007153</td>
<td>2.618805</td>
<td>0.0088</td>
</tr>
<tr>
<td>C(31)</td>
<td>0.161002</td>
<td>0.028417</td>
<td>5.665064</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(42)</td>
<td>-6.55 \times 10^{-5}</td>
<td>0.000163</td>
<td>-0.402840</td>
<td>0.6871</td>
</tr>
</tbody>
</table>
The log likelihood for the system is -461.0617 and the determinant residual covariance for the system is 9.112041.

Turkey, Estimation 2

\[ y(t) = y(t - 1) + c_{14}s(t) \]
\[ \pi(t) = c_{21}\pi(t - 1) + c_{22}(y(t - 1) - y^P) \]
\[ s(t) = s(t - 1) + c_{31}r^C(t) \]
\[ r^C(t) = r^C(t - 1) + c_{42}s^2(t) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Point est.</th>
<th>Std. error</th>
<th>z-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(14)</td>
<td>0.000740</td>
<td>0.000235</td>
<td>3.154050</td>
<td>0.0016</td>
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<tr>
<td>C(21)</td>
<td>0.449603</td>
<td>0.160149</td>
<td>2.807401</td>
<td>0.0050</td>
</tr>
<tr>
<td>C(22)</td>
<td>0.043063</td>
<td>0.015883</td>
<td>2.711301</td>
<td>0.0067</td>
</tr>
<tr>
<td>C(31)</td>
<td>0.173568</td>
<td>0.030449</td>
<td>5.700282</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(42)</td>
<td>-1.13 \times 10^{-8}</td>
<td>0.000157</td>
<td>-0.071937</td>
<td>0.9427</td>
</tr>
</tbody>
</table>

The log likelihood for the system is 326.7167 and the determinant residual covariance for the system is 7.89 \times 10^{-10}.

While we employ a variety of empirical estimates for each nation (and are publishing only a handful of the tightest fits to the data), a broad pattern emerges. The empirical models yield, among other things, point estimates that can be used to solve for the equilibrium values of the two equilibria implied by the model. In all cases (both those listed here and those unlisted), the two equilibria yielded by the model comprise a “full-employment” equilibrium consistent with the overall pattern of the data, and an “underemployment” equilibrium where real GDP is between zero and 1 monetary units of the nation, while the other variables (inflation, the exchange rate, and the credit market interest rate) take on unrealistic values (such as -4,000 per cent). In essence, the second equilibrium is trivial and devoid of economic content, and thus we are forced to reject it as a plausible outcome.

With each of the models yielding a single viable equilibrium and a second meaningless equilibrium, we conclude that we cannot find evidence for the existence of multiple equilibria, either in the data of nations that had experienced recent sizable currency crises or in the data of nations that had
not. Intriguingly, although this result agrees with a sizable amount of empirical analysis on the subject, particularly by Dagsvik and Jovanovic (1994), it stands in contrast to some of the theoretical, third-generation DSGE models of currency crisis, such as by Cooper and Corbae (2002).

4.4 Cutting-edge econometrics

The econometric study of multiple-equilibrium models is undergoing a revolution. Most of the advances are coming from microeconomics, where game-theoretic models often have multiple equilibria and require econometric verification of their results. Formerly, it was believed that such models could not be properly studied in an empirical light, and that therefore game-theoretic models could not truly call themselves “scientific.” This concern is being addressed at a theoretical level, yet many of the results are not fully ready for wide-scale implementation. The broad principle underlying many of the proposed solutions to this problem is as follows: The problem of estimating a multiple-equilibrium model can be reduced to estimating a single-equilibrium model provided that the equilibrium-selection mechanism is estimated simultaneously with the rest of the model.

As an example, Gregoir (2002) identifies the duality between multiple-equilibrium models and the identification principle, and identifies issues common to both areas of research, while offering a variety of possible econometric solutions to the problems cited, both of a parametric and a semi-parametric nature. The most promising research in this area, not yet published, is by Bisin, Moro, and Topa (2002), who are attempting to assemble a broad theoretical justification for a series of empirical techniques they have pioneered in recent years. Two published examples of this recent empirical work are Moro’s (2003) study of race-based statistical discrimination and wage inequality and Bisin, Topa, and Verdier’s (2004) study of socialization and the probability of various types of religious intermarriage. The work in this area is ongoing, with more results yet to be published.

5. Conclusions

In this paper, we use the general framework of standard AS-IS models commonly employed by monetary policy-makers to search for evidence of multiple equilibria in the data of nations that had and had not experienced recent
currency crises. We find no evidence to support the existence of multiple equilibria. While the obvious conclusion is that there are no multiple equilibria, there are at least two plausible alternative hypotheses that we are not able to preclude. First, it is possible that our model, while generally a good guide for monetary policy-making, is insufficient to capture the true essence of multiple equilibria. Second, it is also possible that our econometric analysis is not sophisticated enough to solve the vexing challenges posed by models of multiple equilibria.

Without evidence of multiple equilibria, we conclude that there should be no additional bias to the overall direction of monetary policy imposed by the possibility of currency crisis. That is, without the possibility of excessively tight or loose monetary policy pushing the economy into a dangerous path towards a permanent, low-output state, policy-makers should not fear either permanently damaging the promotion of full employment or unintentionally causing or prolonging a currency crisis. The usual rules of prudent use of policy instruments apply without further proviso.
References


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