What Can the Life Cycle Model Tell Us About 401(k) Contributions and Participation?

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Abstract

This paper solves and simulates a stochastic life cycle model of an economy with 401(k) plans. We use the model to establish a benchmark for patterns of contributions and participation and show how these patterns depends on such features as employer matching, vesting policies, and the availability of plan loans. Consistent with empirical studies, the model predicts relatively low participation rates among younger workers and shows that these rates tend to rise with more generous matching, lower vesting periods, and the availability of loans.

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1 Introduction

By any measure, the growth of 401(k) plans over the last twenty years has been impressive. From 1984 to year-end 2003, for example, the number of participants increased from 7.5 to 42.4 million, while plan assets rose from \$91.75 billion to \$1.9 trillion.[1] One of the consequences of this rapid growth is that workers increasingly shoulder more responsibility for their own retirement saving, choosing whether to participate in the plans, how much to contribute, and whether to take early withdrawals. In many respects, this flexibility can be viewed as an improvement over traditional pensions. The portability of the plans enables workers to leave jobs without forfeiting most of their pension savings, employees can elect a path of saving that suits their individual life cycle needs, and asset allocation decisions can be tailored to individual preferences over risk and return. Nevertheless, this flexibility has fueled concern that for a variety of reasons workers might fail to save adequately for retirement. Saving decisions are complicated, and some employees might lack the financial sophistication to make sensible choices about participation and contributions.

Critics concerned about inadequate 401(k) savings often point to low participation rates as evidence of irrational undersaving.[2] The presumption is that, given the advantages of tax-deductibility and employer matching, most workers should opt into the plan. But many do not. In 2001, for example, just 74 percent of all eligible workers participated. The numbers for younger workers are even lower; less than 66 percent of individuals aged 20 to 40 who earned between \$20,000 and \$40,000 participated in the plan.[3]

Low participation rates may be the product of rational decisions on the part of income-constrained workers, or they may be due to myopic decision making, incomplete information, or a failure to understand the implications of different saving choices for retirement income. All may be factors, but without first analyzing the case of rational decision making, it is difficult to measure the extent to which the saving behavior we observe is suboptimal. The goal of this paper is to provide a benchmark characterization of optimal contributions and participation. To accomplish this, we solve and simulate a stochastic life cycle model that includes important 401(k) plan features such as vesting, employer matching, plan loans, and contribution limits.

Our approach builds on previous stochastic life cycle models of retirement accounts (Engen, Gale, and Scholz, 1994; and Laibson, Repetto, and Tobacman, 1998) by examining the importance of various 401(k) features for optimal contribution and participation decisions. The structure of our model is intended to capture the fundamental tradeoff associated

[1] Source: Employee Benefit Research Institute (2005a).

[2] Studies finding participation behavior that is at odds with traditional economic models include Hinz and Turner (1998), Bernheim (1998), Clark and Schieber (1998), and Choi, Laibson, and Madrian (2004).

[3] Participation numbers are taken from Munnell and Sunden (2002). They calculate participation rates using data from the 2001 Survey of Consumer Finances.

with 401(k) contributions—higher returns vs. lower liquidity—and the dependence of this tradeoff on the nonlinearities induced in people's budget sets by vesting rules, withdrawal penalties, matching limits, and contribution limits. Individuals in our model can place savings in either a tax-sheltered 401(k) plan or a conventional savings account. Assets in the 401(k) earn a higher after-tax rate of return, but they are substantially illiquid due to the restrictions and penalties on pre-retirement withdrawals. Liquidity is important in our model because risk-averse individuals have an incentive to smooth consumption over earnings and employment shocks. We analyze the relative importance of returns and liquidity by numerically solving for consumption and contribution decision rules, which we then use to simulate thousands of life histories for individuals that differ by education and income.[4]

We draw the following conclusions from our simulations. First, the baseline specification predicts low participation rates for younger workers that are consistent with what we observe in the data, with rates ranging from 24 to 56 percent for individuals aged 20 to 29 and from 75 to 87 percent for individuals aged 30 to 39. We find, however, that the model overpredicts participation for ages 40 to 64 by about 20 percentage points. Second, we find that plan features such as vesting, matching, and loans have a substantial effect on optimal saving and participation decisions. Allowing for plan loans, for instance, increases the participation rate of younger college graduates by 30 percentage points. Increasing the match rate also boosts participation, but it appears to have the opposite effect on the average contributions of middle-aged workers. Compared with matching and loans, vesting has a smaller effect, but it can still reduce the participation rates of the young by as much as 15 percentage points. Our third finding is that the effects of these features depend critically on assumptions about risk aversion and the elasticity of intertemporal substitution. In general, we find that more risk averse individuals tend to be less responsive to plan incentives.

The simulation results are generally consistent with previous empirical studies on participation and contributions. Kusko, Poterba, and Wilcox (1998) examine plan data from a firm and find that participants are strongly influenced by the nonlinearities induced by the 401(k) matching and contribution limits. These limits play an important role in our simulations as well, which indicate that individuals spend an average of between 5 and 15 years contributing within 95 percent of the matching limit.[5] Bassett, Fleming, and Rodrigues (1998) examine participation using the 1993 Current Population Survey and find that it rises with income, age, tenure, the presence of matching, and home ownership. With the exception of home ownership, which we do not model, our simulations show similar

[4] In the paper, the terms "education," "dropout," "high school graduate," and "college graduate" refer to a specific set of assumptions about the structure of earnings and employment risk that we discuss in Sections 4.1 and 4.2. They should carry no further connotations, such as, for example, financial sophistication.

[5] We cannot explain, however, another finding of theirs that workers exhibit considerable inertia in their contribution decisions.

predictions.[6] Two studies—one using plan-level data (Papke, 1995) and the other using survey data (Papke and Poterba, 1995)—find that the existence of employer matching has a strong positive effect on participation, but that the effect on contributions is rather weak. Our simulations reproduce this finding, which we interpret as evidence for the countervailing influence of the substitution and income effects of the match rate. Finally, Munnell, Sundén, and Taylor (2000) estimate participation and contribution equations using data from the 1998 Survey of Consumer Finances. In addition to confirming previous studies' findings about the importance of the existence of matching, they find that the availability of plan loans has a positive and significant effect on contributions.[7] This is consistent with our simulation results, which indicate that the ability to access 401(k) savings during employment increases participation and contributions of younger workers.

The rest of the paper proceeds as follows. Section 2 discusses details of 401(k) plans, such as matching policies, contribution limits, and withdrawal penalties, that are relevant to the model. Section 3 presents the model. Section 4 describes the estimation of the earnings processes and other parameters. Section 5 presents the simulation results, and Section 6 concludes.

2 401(k) Rules and Plan Design

2.1 Employer matching limit, contribution limits, plan loans, and withdrawal penalties

The incentive effects of 401(k)s are shaped by the various limits and penalties associated with the plans.[8] Two different types of limits apply to 401(k)s: an employer matching limit, set by the employer, and contribution limits established by law. The matching limits typically range between 1 percent and 10 percent of salary, with a modal value of 6 percent. Because 401(k) contributions are tax-deferred, the Internal Revenue Code limits the amount of employer and employee contributions. The maximum amount an employee can contribute in a given year is determined by the elective contribution limit, equal to $15,000 in 2006 and set to be indexed for inflation thereafter.[9] In addition to the elective contribution limit,

[6] Table 3 of their paper shows that 26.3% of workers with family incomes less than $15,000 participate in plans without matching, whereas 38.6% of such workers participate in plans that offer matching. The corresponding increase in participation for workers with family incomes between $50,000 and $75,000 is from 68.1% to 73.2%. While the existence of matching appears to be important for participation, they find no evidence that the *level* of matching is a factor. The simulations in our paper indicate that matching is more important for the participation of lower-income workers, but in these simulations, the level of the match does make a difference.

[7] They do not measure the effect of loans on participation because the SCF does not include information on the plan characteristics of individuals who do not participate.

[8] For a discussion of the evolution of defined contribution plan characteristics see Mitchell (1999).

[9] Catch-up provisions allow people older than 50 to contribute an additional $5,000 per year.

there is also a limit on total contributions to the 401(k), including any employer matching and employee after-tax contributions. The 2006 limit is the lesser of $44,000 and 100 percent of salary and is scheduled to rise with inflation.

Because 401(k) plans are designed to promote retirement saving, the IRS rules prohibit or penalize most withdrawals made before the age of 59 1/2, with the following exceptions: the individual dies or becomes disabled, the individual is at least age 55 and terminates employment, or the individual terminates employment and takes substantially equal payments from the 401(k) over the expected life of the individual or the joint life of the individual and spouse. Penalized preretirement withdrawals are allowed under the following conditions. First, in the event of a job separation, the individual can make a withdrawal from the 401(k) by paying a 10 percent penalty and income taxes on the amount withdrawn. Second, without terminating employment, an individual can make a penalized "hardship" withdrawal for qualified medical expenses, the purchase of a primary residence, tuition expenses, or preventing eviction or foreclosure. These withdrawals are penalized at 10 percent and subject to current income taxes. Finally, another way to access 401(k) savings before age 59 1/2 is to take out a loan, where the loan cannot exceed the lesser of 50 percent of plan assets and $50,000. When available, plan loans considerably reduce the liquidity costs associated with 401(k) contributions. While not all plans allow borrowing, most of them do. Holden and VanDerhei (2005) report that 87 percent of participants in 2004 were in plans that allowed borrowing. Of these participants, they find that 19 percent had outstanding balances and that these balances amounted to an average of $6,946.

Matching policies, contribution limits, and withdrawal rules introduce nonlinearities in the optimal saving problem that can affect decisions in complex ways. The matching policy of a firm consists of a match rate (as a fraction of contributions) and a match limit (as a fraction of salary), and both of these are important for understanding contribution behavior.[10] Similar to the textbook decomposition of interest rate changes, matching has both an income and a substitution effect, and they work in opposite directions. On the one hand, higher match rates increase the relative return in the 401(k) and therefore tend to increase desired contributions. On the other hand, they also reduce the amount of saving needed to acquire the same level of wealth and therefore to lower desired saving in the plan. Naturally, the net effect of matching will depend on the relative sizes of these income and substitution effects.

The employer matching limit and the elective contribution limit each play an important role in contribution decisions. Part of the opportunity cost of foregoing matching in a given period is the potential inability to make up for it in later periods with increased

[10] In a recent survey (Investment Company Institute, 2000), matching ranks as the second most compelling reason to invest in the plan, behind only "concern over retirement needs."

contributions. That is, contributing less in any given period makes it more likely that desired saving will exceed the matching limit at some point in the future. The matching limit therefore provides individuals with an incentive to even out contributions over time. The elective contribution limit introduces a similar distortion.[11] In this case, individuals have an incentive to smooth contributions in order to avoid facing a binding contribution limit in the future.

2.2 Vesting in 401(k) plans

The law requires that employee 401(k) contributions vest immediately but allows delayed vesting—subject to some limitations—for employer matching. Vesting policies for matching contributions vary across plans and take one of three forms: immediate vesting, graduated vesting, or cliff vesting. With graduated vesting, the employee takes possession of an increasing portion of the matching contributions with each year of tenure.[12] An example of such an arrangement is a plan that vests 20 percent per year for 5 years. Under cliff vesting, the employee is not entitled to any employer matching contributions until a service requirement is met.[13] All three types of vesting policies are common. According to data from the 2000 National Compensation Survey, conducted by the Bureau of Labor Statistics, 25 percent of 401(k) plans held by all private industry workers had immediate vesting, 23 percent had cliff vesting and 46 percent had graduated vesting.[14] Of those plans with cliff vesting, over 90 percent required 3 or more years of service. And of those that offered graduated vesting, more than 70 percent did not fully vest before 5 years. Even though these periods will probably decrease in future years because of the stricter EGTRRA rules on vesting durations, their length suggests that vesting could have a substantial impact on saving behavior.

Employees who leave their job before their employer matching contributions fully vest can miss out on a substantial amount of money. In 1998, for example, 15 percent of all 401(k) participants would have lost a portion of their matching contributions in the event of a job separation.[15] For people that experienced such a separation and had to forfeit

[11] Carroll and Kimball (2001) provide a theoretical framework for analyzing the effect of future constraints on current consumption decisions.

[12] The 2001 Economic Growth and Tax Relief Reconciliation Act tightened the vesting rules for plans with employer matching. Under the new rules, graduated vesting must be completed in 6 years, with 20% after the second year, and an additional 20% in each of the following years until 100% is vested in year 6. Before EGTRRA, plans with graduated vesting for employer matching contributions had to vest fully by 7 years, with 20% vesting each year starting at year 3.

[13] Plans that have employer matching contributions and cliff vesting must fully vest those contributions in a maximum of 3 years. Before EGTTRA, the maximum period for cliff vesting was 5 years.

[14] Source: Table 88 in the U.S. Department of Labor and the Bureau of Labor Statistics's, "National Compensation Survey: Employee Benefits in Private Industry in the United States, 2000." The table can be found on the BLS's web site: http://www.bls.gov/ncs/ebs/sp/ebbl0019.pdf.

[15] Author's calculation from the 1998 Survey of Consumer Finances.

their unvested matching contributions, the average loss amounted to nearly $6,000 (in 1998 dollars), which translates into an average reduction in 401(k) plan savings of 33 percent.

In contrast to defined benefit plans, where individuals exercise little choice over saving decisions, defined contribution plans such as the 401(k) allow participants to strategically time contributions in order to minimize the risk of losing unvested contributions. Such a strategy must balance a set of tradeoffs. The main benefits of contributing to the plan consist of employer matching and the tax deferral on both contributions and investment earnings. The primary cost is the reduced liquidity associated with the withdrawal rules on the 401(k).

At first glance, it is not obvious that vesting should have a substantial impact on a person's 401(k) contributions. After all, none of the employee's contributions are lost with a job separation, and even though the employer matching contributions may be lost with some probability, the expected value of employer matching is still quite high for typical matching arrangements. To see how vesting can still affect a participant's saving decisions, consider the case of a person who saves primarily as a precaution against shocks to labor income (e.g., a person in the early stage of the life cycle who, in the absence of risk, would like to borrow). In order to avoid the possibility of losing any employer matching contributions, this person could delay contributing until she is fully vested in the plan.

What are the costs of such a strategy? First, there is the difference in the compounded pre-tax investment returns on employee contributions to the 401(k) plan and the returns in the taxable account. Second, this difference in the compounded investment returns also applies to the forgone matching if the person places the delayed contributions plus any taxable returns in the 401(k) plan after vesting. Third, there are forgone future investment earnings on the difference in 401(k) savings associated with nondelayed and delayed contributions. Finally, delaying contributions can be costly because it makes it more likely that the person will miss out on employer matching contributions in the future. This last cost would not arise in the absence of limits on matching or contributions because missed match opportunities could always be made up in the future by shuffling savings from the taxable account into the 401(k) plan. With limits, however, a person who delays contributing to the plan will have an increased probability of desiring contributions in excess of either the employer matching limit or the annual contribution limit in future years.

In order to get a sense of how large these costs might be, consider a simple example of a person trying to decide whether to contribute $100 today to a 401(k) plan that will vest in t_v years, or instead wait to contribute until the plan is fully vested. Assume that the tax rate τ on ordinary income remains constant over time and that the after tax return on taxable investments is given by $r(1-\tau)$. Finally, assume that there are no limits on matching or contributions. What is the cost of forgoing the $100 contribution with 100%

matching?

If the person makes the \$100 contribution today, it will be worth

$$\$200(1+r)^{t_v}$$

at the point of vesting. If, on the other hand, the person waits until vesting, she must pay income taxes on the \$100 before placing it in the taxable account. At the time of vesting, this amount would then be worth

$$\$100(1-\tau)[1+r(1-\tau)]^{t_v}.$$

Suppose now that the individual decides to use this amount to finance a pre-tax matched contribution to the 401(k). How large would this contribution be? Letting x denote the size of this contribution, the tax deductibility of the plan implies that the contribution will cost the individual $x(1-\tau)$ dollars. Setting this amount equal to the equation above and solving for x, we see that the individual can finance a contribution of

$$x = \$100[1+r(1-\tau)]^{t_v},$$

which with 100 percent matching would generate total contributions of

$$\$200[1+r(1-\tau)]^{t_v}.$$

The difference between the two strategies at the time of vesting is then given by

$$\$200(1+r)^{t_v} - \$200[1+r(1-\tau)]^{t_v}.$$

If the person contributes an additional amount equal to this difference at time t_v, then the income at retirement will be identical under the two strategies. If she does not contribute this difference, then the difference between the amounts upon taxable withdrawal at time t_r would equal

$$\begin{aligned} & (1-\tau)\$200(1+r)^{t_v}(1+r)^{(t_r-t_v)} - (1-\tau)\$200[1+r(1-\tau)]^{t_v}(1+r)^{(t_r-t_v)} \\ = \; & (1-\tau)\$200(1+r)^{t_r} - (1-\tau)\$200[1+r(1-\tau)]^{t_v}(1+r)^{(t_r-t_v)}. \end{aligned}$$

The first term equals the initial contribution plus matching, compounded for t_r years, minus taxes paid upon withdrawal. The second term is smaller because the returns are compounded at the after-tax rate of $r(1-\tau)$ for the first t_v years.

For a tax rate τ of 20 percent, Table 1 shows how large these costs would be for a person who plans on withdrawing the funds in 20 years. The table displays the costs at the vesting period t_v, the withdrawal period t_r, the present value of these costs discounted at the pre-tax rate of return (the present value is the same for both t_v and t_r since after t_v, the amounts accumulate at the rate of discounting), and the costs as a percent of the amount at vesting. Even for relatively long vesting periods these costs are modest, and discounted at the rate of return, they are even smaller. For example, with a return of 3 percent, the discounted cost of waiting to reach a vesting period of 4 years is $4.62. In percentage terms, the losses range from 0.58 to 3.57 for a return of 3 percent, and from 0.96 to 5.90 for a return of 5 percent.

Given the liquidity benefits of delaying contributions and the possibility of losing the employer matching contributions, the numbers from the example above suggest that it is easy to rationalize a strategy of waiting. But of course, this simple example ignores many aspects that are important to the life cycle problem, including the time path of earnings, a precautionary saving motive, and uncertainty about earnings, mortality and employment. The purpose of solving the more complicated model is to see how these features affect the contribution decisions of people in a more realistic economic environment.

3 The Model

This section develops a life cycle model that provides a framework for investigating the effects of particular 401(k) plan features, such as vesting, matching, and plan loans, on plan participation and contributions. The basic structure of the model resembles those in three previous life-cycle papers on retirement saving: Engen, Gale, and Scholz (1994), Laibson, Repetto, and Tobacman (1998), and İmrohoroğlu, İmrohoroğlu, and Joines (1998). The model in this paper extends these by introducing vesting and the possibility of job separation that makes vesting potentially costly for 401(k) plan participants.

People in the model make consumption and saving decisions that maximize their expected lifetime utility, where saving can be placed either in a conventional saving account or a tax-sheltered 401(k) plan. Both saving locations earn the same constant pre-tax rate of return, but contributions to the 401(k) are tax deductible and returns inside the plan accrue free of tax. These tax benefits, along with employer matched contributions, provide strong incentives to save in the plan. The main reason people choose to save outside of the 401(k) plan is liquidity. Earnings are uncertain, and in the baseline version of the model, borrowing is not allowed. Since the 401(k) plan rules penalize or prohibit most pre-retirement withdrawals, people may prefer to smooth earnings fluctuations using savings built up in the unsheltered account. These important tradeoffs between liquidity and return are characterized in the model described below.

Preferences The model economy consists of three types of people—high school dropouts, high school graduates, and college graduates—who differ in their earnings profiles, employment risk, and income uncertainty. Lifespans are uncertain but last no longer than T years. Let ϕ_t denote the unconditional survival probability of living to age t. The conditional survival probability is then given by $p_t = \phi_t/\phi_{t-1}$. Each type of person shares the same constant relative risk aversion preferences, given by

$$U_t = E_t \sum_{i=t}^{T} \frac{\phi_i}{\phi_t} \beta^{i-t} u\left(c_i\right) \tag{1}$$

$$u\left(c_t\right) = \begin{cases} \frac{c_t^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln(c_t) & \text{if } \sigma = 1, \end{cases}$$

where β is the discount factor, c_t is consumption in year-2000 dollars, E_t is the expectations operator, and σ is the coefficient of relative risk aversion. (The inverse of σ is the intertemporal elasticity of substitution. As we will see in the simulations, this parameter plays an important role in determining precautionary saving behavior, as well as the optimal path of consumption over the life cycle.) Working life begins at age 20, retirement occurs at 65, and the maximum lifespan is 85. We adopt the timing convention that variables with the age subscript t refer to the beginning of that age period.

Earnings process During the working years, both employment and earnings are uncertain. Let emp_t be an indicator that equals 1 if the individual is employed, and 0 otherwise. Dropping the individual subscript i, the natural logarithm of earnings at age t is given by:

$$\ln(y_t) = emp_t \left[g(\mathbf{z}_t) + \eta_t\right] + (1 - emp_t)UI \times g(\mathbf{z}_t), \tag{2}$$

where

$$\eta_t = \rho\eta_{t-1} + \xi_t, \quad \xi_t \sim iid(0, \sigma_\xi^2). \tag{3}$$

Log earnings during employment consist of a trend component $g(\mathbf{z}_t)$ that depends on a list of individual-specific variables, $\mathbf{z}_t$, such as age, household size, and race, as well as an AR(1) shock. If the individual is unemployed, she receives an amount of income equal to the replacement rate of unemployment insurance, UI, times her trend earnings. The probability of entering unemployment ($\Pr(emp_t = 0)$) depends on age and education but is assumed to be independent of the previous period's state of employment. The estimated earnings profiles, the covariance structure of the AR(1) process, and the employment probabilities will be discussed in the Section 4.2 below.

Vesting The model allows for both cliff vesting, where nothing vests until the duration is met, and graduated vesting, where a fraction of employer contributions vests each period. Let α_t denote the fraction of matched contributions that vests in period t, and d denote the duration at which the employee is fully vested into the 401(k) plan. Under cliff vesting, the vesting fraction is given by

$$\alpha_t = \begin{cases} 1 \text{ if } ten_t \geq d \\ 0 \text{ if } ten_t < d, \end{cases} \tag{4}$$

where ten_t is the person's tenure, equal to 0 during the first year of employment and increasing by 1 thereafter. With graduated vesting, the fraction is given by

$$\alpha_t = \frac{ten_t}{d}. \tag{5}$$

So, for example, under a five-year vesting schedule, 20 percent vests after the first year, 40 percent after the second, and so on.[16]

Savings and 401(k) accounts In order to make the problem tractable, we assume that each account shares the same before-tax real rate of return R (the after-tax rates of return differ). Vested assets in the 401(k) plan accumulate according to

$$a_{t+1} = R\{a_t + x_t + \alpha_t \left[m_t + \mu \min(x_t, \psi y_t)\right]\}, \tag{6}$$

where x_t is the employee's contribution to the 401(k) plan, m_t is the amount of the employer's matching contributions that did not vest the period before, y_t is income, μ is the employer match rate, ψ is the maximum fraction of income the employer will match, and α_t is the fraction of the matched contributions that vests in period t. Unvested matched contributions evolve as follows:

$$m_{t+1} = \begin{cases} R(1-\alpha_t)\left[m_t + \mu \min(x_t, \psi y_t)\right] & \text{if } emp_t = 1 \text{ and } x_t \geq 0 \\ R(1-\alpha_t) m_t & \text{if } emp_t = 1 \text{ and } x_t < 0 \\ 0 & \text{if } emp_t = 0 \text{ or } t \geq 64. \end{cases} \tag{7}$$

Equation (7) reflects the property that any unvested portion of the matched contributions is lost in the event of a job separation. In the case of cliff vesting, α_t will be equal to zero if tenure is below the vesting duration and equal to one otherwise. Once the employee is vested, matches enter immediately into the 401(k) account, and $a_{t+1} = R[a_t + x_t + \mu \min(x_t, \psi y_t)]$.

[16]Common vesting arrangements combine elements of equations (4) and (5) so that a graduated vesting schedule often starts only after one or two years of tenure. We do not consider such hybrid arrangements because doing so would require solving many more versions of the model, and each version takes as long as two days to solve.

The evolution of assets in the unsheltered account is given by

$$s_{t+1} = R(s_t + y_t - c_t - x_t - taxes_t - pen_t), \tag{8}$$

where $taxes_t$ and pen_t denote respectively taxes and the early withdrawal penalty. The early withdrawal penalty function is given by

$$pen_t = \begin{cases} -\kappa(emp_t)\min(0, x_t) & \text{if } t < 65 \\ 0 & \text{if } t \geq 65, \end{cases}$$

where $\kappa(emp_t)$ is the penalty rate on early withdrawals made when the employment state is emp_t. In all parameterizations of the model, $\kappa(0)$ is 10 percent, but $\kappa(1)$ is allowed to vary. In the absence of plan loans and provisions for hardship, $\kappa(1)$ should theoretically be equal to 1, reflecting the fact that in this case, the plan is perfectly illiquid during employment. If, on the other hand, we want to consider plan loans or other sources of liquidity, then $\kappa(1)$ should be less than 1. In addition to the $\kappa(1) = 1$ case, we also solve the model under the assumption that withdrawals during employment are costless, or $k(1) = 0$. Note that this specification of the withdrawal penalty is only meant to capture the fact that plan loans make 401(k) plans considerably more liquid.[17]

Note that it is possible to combine equations (6) and (8) in order to obtain the following total resource constraint.

$$c_t + \underbrace{(a_{t+1} + s_{t+1})/R - (a_t + s_t)}_{\text{total saving}} = \underbrace{y_t + \alpha_t [m_t + \mu \min(x_t, \psi y_t)]}_{\text{income + matching}} - taxes_t - pen_t.$$

The resource constraint requires that the present value of consumption and saving must equal total income, including any employer matching, net of taxes and penalties.

3.1 Individual's Problem

At the beginning of period t, the individual observes her employment status, mortality status, and earnings. The individual then decides how much to consume, how much to contribute to the 401(k) account, and how much to save in the unsheltered account. The last decision—how much to save in the unsheltered account—is determined by the first two choices.

The set of state variables for the problem is $\Omega_t = \{s_t, a_t, m_t, \xi_t, emp_t, ten_t, t\}$. The control

[17] We do not model plan loans more explicitly because doing so would require carrying around an additional state variable indicating the amount of outstanding balances.

variables are $\{c_t, x_t\}$. The value function for the consumer's problem is given by

$$\begin{aligned} V_t(\Omega_t) &= \max_{c_t, x_t} \{u(c_t) + \beta p_{t+1} E_t [V_{t+1}(\Omega_{t+1})]\} \\ &\quad \text{subject to} \\ s_{t+1} &= R(s_t + y_t - c_t - x_t - taxes_t - pen_t) \\ a_{t+1} &= R\{a_t + x_t + \alpha_t [m_t + \mu \min(x_t, \psi y_t)]\} \\ m_{t+1} &= \begin{cases} R(1-\alpha_t)[m_t + \mu \min(x_t, \psi y_t)] & \text{if } emp_t = 1 \\ 0 & \text{if } emp_t = 0 \end{cases} \\ s_t &\geq 0,\ a_t \geq 0 \\ x_t &\leq L \text{ if } t < 65 \\ x_t &\leq 0 \text{ if } t \geq 65. \end{aligned} \tag{9}$$

The expectation in equation (9) is taken over uncertainty about employment and earnings. Specifically,

$$\mathbb{E}_t [V_{t+1}(\Omega_{t+1})] = \sum_{i=0,1} \Pr(emp_{t+1} = i) \int V_{t+1}(\Omega_{t+1}) dF(\xi_{t+1}|\xi_t), \tag{10}$$

where $F(.)$ is the distribution of the earnings shock. In practice, we integrate the expectation in equation (10) using a two-state Markov approximation of the AR(1) process in equation (3).[18]

It is worth emphasizing that our model describes the choices and contraints faced by individuals who are, and have always been, eligible for a 401(k) plan. In reality, however, many individuals work for firms that either do not offer 401(k) plans or offer hybrid plans that incorporate features from both defined benefit plans and defined contribution plans. While these other arrangements would be interesting to study in their own right, our model focuses on characterizing the behavior of the large number of 401(k)-eligible workers.

The institutional features of the 401(k) that make the plan interesting to study also introduce complexities in the computational solution. The appendix discusses the solution method. The basic approach involves using discrete space methods and interpolation to solve the model from the last period backwards. At each time period, the value function and corresponding decision rules are computed at a predetermined set of points in the state space. We then use cubic spline interpolation between points and extrapolation beyond points in order to construct functions that are defined at all points in the state space. These interpolated functions are then used in the next period's optimization problem. Starting

[18]The quadrature method follows Tauchen and Hussey (1991).

with period T, equation (9) is solved recursively to yield consumption and saving functions, which take tenure, earnings, the size of the unvested match, and asset levels in each account as their arguments.

4 Data and Parameters

This section describes the estimation and selection of parameters in our model. We do not adopt the calibration approach used in much of the real business cycle literature (e.g., choosing parameters to match empirical wealth-to-income ratios) because the model economy does not correspond to the actual economy in at least one important respect: The model assumes that individuals have had access to 401(k) plans for their entire life cycle, whereas participation in the plan did not begin in earnest until the early 1980s. As such, the wealth-to-income ratio generated by our model should not necessarily match the ratio in the data. Instead of calibration, our approach is to choose commonly-used parameter values and perform a sensitivity analysis to test for robustness.

4.1 Earnings

The institutional features of 401(k) plans that we model, such as contribution limits, matching rules, and vesting policies, apply to individuals rather than households. We therefore estimate earnings regressions and their associated covariance structures at the level of the individual. Using an unbalanced panel of data from the 1980-1997 waves of the Panel Study of Income Dynamics, we first estimate separate log earnings regressions for workers and retirees by education category. Workers are defined as male household heads aged 20 to 69 who work more than 1,000 hours in a year, earn between \$1,000 and \$1,500,000 (in year-2000 dollars), and list their primary activity to be "working now." Individuals are considered to be retired if they are older than 55, work fewer than 500 hours in a year, and list their primary activity to be "not working now." In order to maintain a representative sample, we drop observations from the PSID's oversampling of lower-income families in its Survey of Economic Opportunity. The sample for workers contains 12,641 observations for college graduates, 27,555 observations for high school graduates, and 6,712 observations for dropouts. For the retirement sample, the number of observations is 8,262 for college graduates, 18,989 for high school graduates, and 20,332 for dropouts.

We estimate the following equation for workers using OLS on the population-weighted PSID data.

$$\ln(y_{i,t}) = \beta_0 + \mathbf{age}_{i,t}\beta_1 + \beta_2 race_i + \beta_3 hhsize_{i,t} + \beta_4 UE_{i,t} + \mathbf{byear}_{i,t}\beta_5 + \varepsilon_{i,t}, \tag{11}$$

where $\ln(y_{i,t})$ is the natural logarithm of earnings of individual i at year t, $\mathbf{age}_{i,t}$ is a polynomial of degree 3 in age, $race_i$ is the individual's race,[19] $hhsize_{i,t}$ is the number of people in the household, $UE_{i,t}$ is the unemployment rate in the state where the individual resides at time t, and $\mathbf{byear}_{i,t}$ is a set of 5-year cohort dummies spanning the period 1910 to 1981. We include the state unemployment rate,[20] which is meant to proxy for time-specific business cycle fluctuations, because it is generally not possible to control simultaneously for age, year, and cohort effects.[21] Even though the dependent variable is individual earnings, we control for household size because individuals with larger families might differ systematically in hours worked and wages.

We assume that labor earnings are zero in retirement. In order to construct income profiles for retirees, we estimate an equation similar to equation (11) using log retirement income (Social Security retirement benefits plus defined benefit annuity payments), instead of log earnings, as the dependent variable. Retirement income is reported only at the household level. Because we are interested in individual income, we construct the retirement income profiles assuming that the household size is equal to 1.

The estimation results for equation (11) are reported in Table 2. The sign pattern on the age polynomial (positive, negative, positive) reflects the well-known hump shape of the earnings profiles. These profiles can be seen in Figure 1, which displays expected earnings by education group for individuals with characteristics (i.e., race, household size, state unemployment rate, and birth year) set equal to their sample means. Two features stand out in the figure. First, the levels of lifetime earnings differ substantially across education groups, with college graduates earning much more than the other two groups. Second, the curvatures of the profiles are also quite different. Compared with the other two groups, college graduates have a much bigger hump in their profiles, as well as a lower replacement rate of income in retirement.

With the coefficient estimates of equation (11) in hand, we then turn to estimating the covariance structure of the predicted residuals.[22] We assume that the residuals, $\varepsilon_{i,t}$, can be decomposed into a fixed effect, a transitory effect, and an AR(1) component. Specifically,

$$\varepsilon_{i,t} = \alpha_i + \eta_{i,t} + \upsilon_{i,t}, \tag{12}$$

[19]Because race does not vary with time, it could equivalently be treated as part of the fixed effect in the error structure described below.

[20]Unemployment rates are taken from the Local Area Unemployment Statistics program at the Bureau of Labor Statistics (http://www.bls.gov/lau).

[21]Gourinchas and Parker (1997) and Laibson, Tobacman, and Repetto (1998).

[22]The methodology for estimating the covariance structure follows the literature on the dynamics of earnings, which includes Lillard and Willis (1978), MaCurdy (1982), Chamberlain (1984), Abowd and Card (1989), and Dickens (2000).

where

$$\eta_{i,t} = \rho\eta_{i,t-1} + \xi_{i,t},$$

where $\upsilon_{i,t} \sim iid(0, \sigma_\upsilon^2)$ and $\xi_{i,t} \sim iid(0, \sigma_\xi^2)$ are white noise error terms.

Following Abowd and Card (1989) and others, we estimate the parameters σ_υ^2, σ_ξ^2, and ρ using the Generalized Method of Moments (GMM) to minimize the distance between the theoretical and empirical autocovariances. Equation (12) implies the following structure for the theoretical autocovariances.

$$E(\Delta\varepsilon_{i,t}\Delta\varepsilon_{i,t-j}) = \begin{cases} \frac{2\sigma_\xi^2(1-\rho)}{1-\rho^2} + 2\sigma_\upsilon^2 & j = 0 \\ \frac{-\sigma_\xi^2(1-\rho)^2}{1-\rho^2} - \sigma_\upsilon^2 & j = 1 \\ \frac{-\sigma_\xi^2\rho^{j-1}(1-\rho)^2}{1-\rho^2} & j > 1. \end{cases}$$

Let $m_j = E(\Delta\varepsilon_t\Delta\varepsilon_{t-j}) - (1/(n_j - 1))\sum_{i=1}^{n_j}\Delta\varepsilon_{i,t}\Delta\varepsilon_{i,t-j}$ denote the jth moment condition, where n_j is the number of observations associated with that moment. Given our 18 years of PSID data, we have $171 = 18(18+1)/2$ possible moment conditions to estimate our three parameters. We minimize the distance between the entire set of theoretical and empirical autocovariances using the procedure discussed in the technical appendix of Dickens (2000), which provides a method for applying GMM to an unbalanced panel.[23] In light of Altonji and Segal's (1994) finding that the optimal minimum distance estimator is severely biased with smaller sample sizes, we opt for an equally-weighted matrix in our minimization routine. The estimated parameter vector $\widehat{\theta} = (\widehat{\rho}, \widehat{\sigma}_\upsilon^2, \widehat{\sigma}_\xi^2)$ minimizes $\mathbf{m}(\theta)'\mathbf{W}\mathbf{m}(\theta)$, where $\mathbf{W}$ is the identity matrix, and $\mathbf{m}(\theta)$ is the $j \times 1$ vector of moment conditions.

Table 2 reports the parameter estimates and standard errors. The estimates of the AR(1) coefficient, ρ, and the variances are all well within the range of previous estimates and imply a fair amount of persistence in the earnings process.[24]

4.2 Other Parameters

In addition to the earnings process, we also need to specify parameters for taxes, employment risk, unemployment insurance, mortality risk, risk aversion, and the discount rate.

The tax structure is based on the 2005 federal personal income tax rates and standard deduction for a single filer. Because income in our model never rises above \$150,000, we need only the first four brackets, \$7,300, \$29,700, \$71,950, and \$150,150, which have corresponding tax rates of 10%, 15%, 25%, 28%, and 33%. The standard deduction is \$5,000.

[23] The author's Matlab code for the GMM estimation is available upon request.

[24] For previous estimates of the AR(1) component of log earnings, see Hubbard, Skinner, and Zeldes (1995), Engen, Gale, and Scholz (1994), and Laibson, Repetto, and Tobacman (1998). These estimates of ρ range from 0.511 in Laibson, Repetto, and Tobacman to 0.955 in Hubbard, Skinner, and Zeldes.

(All values are deflated to year 2000 dollars.)

Job separation in the model is exogenous. We take our measure of involuntary separation from data on job displacement. Specifically, we use the probabilities of displacement from Farber (1997), which reports displacement rates from the Displaced Worker Survey supplement to the Current Population Survey. Farber's numbers correspond to the probability of job loss occurring at least once within a three year time period. We convert these into one-year probabilities by assuming that the rate of displacement is constant across years. Table 3 displays these probabilities for different age and education groups. Looking at the table, we can see that the probability of job loss falls with education and age. In the event of job loss, we assume that individuals receive unemployment insurance equal to 50 percent of trend income.

The assumption that job loss is exogenous understates the likelihood of a worker forfeiting matching contributions. After all, the majority of separations are voluntary, and some workers will decide to leave a job even if doing so means they will lose a portion of their employer match. The extent of voluntary forfeiture is unknown, but evidence on employee tenure, which shows that people repeatedly change jobs over the course of their career, suggests that it could be substantial.[25] Because our model does not allow for the possibility of losing matching contributions through voluntary separation, we argue that it represents a lower bound on the costs of vesting from the employee's perspective.

The demographic and utility parameters are chosen as follows. People live to a maximum age of 85. For all ages under 85, we calculate the survival probabilities using data on men from the Social Security Administration's Period Life Tables, 2001.[26] The coefficient of relative risk aversion (CRRA) can have a large effect on saving since it determines both risk aversion and the willingness to substitute consumption intertemporally. As such, we solve the model for a range of CRRAs (1, 2, and 3) that is consistent with many previous parameter estimates and calibrations.[27] The discount factor is assumed to be 0.967.

We assume that assets held inside and outside the 401(k) grow at the same pre-tax real rate of return of 3 percent. While we make this assumption for computational convenience, it is almost certainly incorrect since an efficient portfolio allocation would place heavily-taxed assets in the 401(k). The returns on the portfolio containing the more heavily-taxed assets should differ systematically from those on the other portfolio containing less heavily-taxed assets.

The 401(k) parameters are chosen as follows. We set the annual contribution limit equal to the 2005 limit of $14,000, adjusted to 2000 dollars. We solve the model for matching of

[25] Source: Employee Benefit Research Institute (2005b).

[26] The table can be found on the SSA's web site: http://www.ssa.gov/OACT/STATS/table4c6.html.

[27] Gourinchas and Parker (2002) estimate the coefficient of risk aversion to be 0.5-1.4, depending on the specification. French (2005) uses a similar technique but finds values in the range of 2.2-5.1.

either 25 percent or 50 percent of earnings up to a matching limit of 6 percent. Plan loans, when available, provide an important source of liquidity in the 401(k). In order to test the importance of this extra liquidity, we solve the model for two extreme assumptions: early withdrawals during employment are either costless or prohibited. The early withdrawal penalty during unemployment is always 10 percent. Finally, in order to show how vesting affects participation and contributions, we solve the model for maximum vesting periods ranging from 0 to 4 years.

5 Results

This section presents the results from the model simulations. The section begins with a look at various life-cycle figures that help establish patterns of 401(k) contribution and participation behavior. The figures show, first, that the majority of saving is done in the 401(k) plan and, second, that contributions and participation follow a distinctive S-shaped path over the working portion of the life cycle, with lower values for younger workers transitioning to higher values for older workers. This latter pattern, we argue, reflects the competing incentives of liquidity and the desire to accumulate resources for retirement. Another pattern that emerges in the figures is the strong influence of contribution and matching limits on saving decisions. The matching limit appears to have the strongest effect in early periods, while the contribution limit becomes more important in the middle years, when earnings reach their peak. Once the basic patterns of behavior have been established, the section then turns to quantifying the effect on these patterns of various plan features, such as vesting, matching, and plan loans. Consistent with empirical studies, the results indicate that these plan features matter most for the saving behavior of liquidity-constrained younger workers.

5.1 Life Cycle Profiles

For each parameterization (values of σ, μ, κ, and d), we generate life cycle profiles (i.e., average paths of variables such as consumption and contributions) by performing 20,000 Monte Carlo simulations of earnings, employment, and mortality shocks. In order to build intuition for the model, we first present an example of just one of the 20,000 simulations. Figure **??** plots a single simulation for a college graduate using our baseline specification ($\sigma = 2$, $\mu = 0.25$, $\kappa = 1$, and $d = 0$). Several features stand out in the figure. First, it is clear that the majority of saving is done in the 401(k) plan, where balances reach almost $400,000 at retirement. Accumulation in the 401(k) does not begin immediately, however, and balances in the plan remain close to zero in the first decade of life. Next, compared with the 401(k), savings in the taxable account are quite low, never rising above $50,000, but there is evidence of precautionary saving early in the life cycle. Finally, savings in both the 401(k) plan and the taxable account increase substantially around age 30. The rapid accumulation of savings after this age tends to smooth consumption since individuals are better able to absorb large fluctuations in earnings.

Figure 2 displays the average profiles corresponding to 20,000 simulations of the baseline specification. We again see that consumption tracks income for the first decade of working life and then tapers off to a relatively smooth path until death. Assets in the 401(k) reach a maximum of about $360,000 at age 64 and then fall rapidly to finance retirement consumption. Savings in the taxable account ("other savings") are considerably smaller and exhibit a distinct bubble pattern from ages 42 to 65. This bubble forms for three reasons. First, since earnings for college graduates reach a peak around age 45, there is an incentive to save in the taxable account in order to maintain relatively high consumption during the period of declining earnings from age 42 to 64. Second, because earnings are relatively high during this period, individuals are also more likely to hit the contribution limit and respond by placing excess desired savings in the taxable account. A third, and related explanation, is that individuals build up taxable savings in order to finance sizable employer-matched contributions even as earnings fall toward retirement. As discussed in Section 2, the matching limit provides a strong incentive to smooth contributions in order to reduce the amount of unmatched contributions.

Figures 3 and **??** display similar profiles for high school graduates and dropouts. High school graduates' 401(k) assets reach $226,000 at retirement, while those for dropouts peak at about $162,000. Compared with college graduates, the consumption profiles of these other education groups do not appear to track earnings for very long in the beginning of the working life. This can be explained by the relative curvatures of the earnings profiles. Younger college graduates experience steeper growth in earnings than the other groups and consequently find themselves more liquidity constrained.

The life cycle profiles give an overview of the simulation results, but they do not provide a detailed look at contribution and saving decisions. Figures 4 and 5 display simulated paths of contributions and taxable saving for employed college graduates and dropouts (the figure for high school graduates resembles the one for dropouts). The figures share the same S-shaped pattern of low contributions early in life followed by a rapid build-up to a plateau extending to retirement.[28] What accounts for this particular pattern of contributions? The following back-of-the-envelope calculation suggests that the lower portion of the "S" is due to the salience of the employer matching limit. Assume that roughly half of the

[28]The bumps in the contribution and saving paths that occur between ages 40 and 64 are due to the discretization of the asset space. We solve the model exactly for different levels of 401(k) assets and other savings, and these levels are more closely spaced for lower values of assets. 401(k) assets grow rapidly during this middle period of the life cycle, and as a result, the interpolated decision rules are somewhat less accurate.

individuals receive a positive earnings shock, while the other half experiences a negative shock. If younger workers contribute to the matching limit when earnings are high but contribute nothing when earnings are low, we would expect younger dropouts, whose high earnings are between $25,000 and $30,000, to contribute $750 ($= 0.5 \times 0.06 \times \$25,000$) to $900 ($= 0.5 \times 0.06 \times \$30,000$). Similarly, high earnings for college graduates are between $60,000 and $70,000, so their contributions should range between $1,800 and $2,100. These predicted contributions are remarkably close to those in Figures 4 and 5. The second plateau of the "S" can be attributed to a combination of progressive tax rates and the contribution limit, each of which provide an incentive to smooth contributions in the years heading into retirement.

One other feature that stands out in the figures is the rise in contributions and decline in other saving toward the end of the working life. Most likely, this is a consequence of the decreasing importance of precautionary balances as individuals approach retirement. The consumption smoothing value of other savings decreases near retirement because individuals know that they can tap their 401(k) balances in a relatively short time period. In contrast, younger workers may be reluctant to draw down savings that can be used to smooth consumption in future periods.

Even though college graduates and dropouts share similar contribution patterns, there are important differences. Whereas younger college graduates save almost exclusively in the taxable account, younger dropouts save in both the taxable account and the 401(k) plan. The difference is again due to the curvature of earnings. For younger college graduates, whose earnings are low relative to future income, the benefits of employer matching apparently do not outweigh the liquidity cost of 401(k) contributions. In contrast, dropouts have a relatively flat earnings profile and therefore face much lower liquidity costs associated with early contributions. Another difference between the figures is that dropouts contribute near the matching limit for much longer (15 years vs. 7 years) than college graduates. One implication of this is that dropouts should potentially be more sensitive to changes in employer matching policies.

Figures 4 and 5 highlight the importance of the matching limit for saving in the taxable account. In the figure for college graduates, other saving remains level at around $500 for the first 10 years, drops sharply as contributions rise to the matching limit, and then rises consistently until age 36 when 401(k) assets increase rapidly. The figure for dropouts shows a similar pattern, with taxable saving increasing until the point of retirement accumulation around age 33. These patterns suggest that in the early portion of the working life conventional savings dominates the 401(k) in the absence of employer matching.

5.2 Contribution and Matching Limits

The results above suggest that the matching and contribution limits play a key role in shaping the optimal path of 401(k) contributions. We characterize the importance of these limits by computing the fraction of workers who contribute above 95 percent of the contribution limit, as well as the fractions contributing below 95 percent, between 95 and 105 percent, and above 105 percent of the matching limit.

Figures 6, 8, and 7 plot these fractions for college graduates, high school graduates, and dropouts. Comparing these figures, it is evident that college graduates spend far less time contributing near the matching limit than do the other two groups. Contributions do not reach the limit until age 29, after which point about half (corresponding to periods with high earnings) hit the limit for the next 7 years. Then, after age 37, the majority of contributions exceed the matching limit, with about 25 percent reaching the contribution limit.

In contrast to college graduates, dropouts begin contributing near the matching limit almost immediately. From ages 21 to 33, between 40 and 50 percent of dropouts' contributions are near the matching limit, with the rest lying below. After that point, most of their contributions are either at or above the matching limit. Almost none of them ever reach the contribution limit.

The pattern of contributions for high school graduates falls somewhere in between those of the other two groups. Matching limit contributions begin at age 25, with contributions rising above the matching limit after age 30. Between ages 37 to 48, where earnings peak, 10 to 20 percent of high school graduates hit the contribution limit.

One interpretation of these results is that the matching limit has a smaller effect on contributions of better educated workers. The steeper profile and higher level of earnings of college graduates make it optimal for these individuals to contribute less in early periods of life when earnings are comparatively low, and then quickly increase contributions above the matching limit as they begin saving for retirement. Because high school graduates and dropouts experience a more gradual change in earnings, they spread their contributions over a larger portion of the life cycle.

5.3 Participation

We now turn to documenting patterns of 401(k) participation. In reality, the participation decision is distinct from the contribution decision, and it is possible for workers to participate in the plan but contribute nothing in a given year. Our model does not make this distinction, so we define participation as positive contributions.

Figure 9 displays participation rates—the fraction of employed workers contributing a positive amount to their 401(k)—for the baseline specification of the model. The participation rate for dropouts climbs steadily from about 50 percent at age 20 to over 95 percent from ages 40 to 64. Participation for high school graduates follows a similar path, but exhibits a stronger spike from 0 to 50 percent in the first year. Combining our findings about the matching limit with the participation results, we conclude that during the early stages of life, individuals participate mostly when earning are high, and when they participate, they contribute to the matching limit.

The average participation rate of college graduates remains close to zero until age 26, after which it rises sharply to close to 50 percent at age 30 and then rises somewhat more gradually to 100 percent at age 40. Participation remains high until close to retirement, at which point there is a slight drop due to the decline in average earnings in these periods.

So far, we have focused on the simulation results for the baseline specification of the model. We turn now to the question of how contributions and participation change in response to different vesting periods, matching rates, withdrawal penalties, and CRRA parameters.

Table 4 displays participation rates by education and age groups for different vesting periods and CRRA parameters. As we would expect from looking at Figure 9, younger workers participate less than older workers, and younger college graduates (aged 20 to 29) participate the least. When $\sigma = 2$ and vesting is immediate, dropouts and high school graduates participate about half the time, while college graduates participate only about one-fifth of the time.

Longer vesting periods tend to decrease participation for all ages, but the effect is strongest in the early working years. For $\sigma = 2$, moving from immediate vesting to 4-year cliff vesting decreases participation by about 4 percentage points for dropouts, 3 percentage points for high school graduates, and 7 percentage points for college graduates. Participation falls in these earlier periods for two reasons. First, because average tenure is low in these periods (tenure is 0 at age 20 and unemployment rates are relatively high) many younger workers have not fully vested in their 401(k) plans. And second, the fact that earnings are relatively low in earlier ages means that the liquidity cost of making 401(k) contributions is especially high. The effect of vesting on participation is comparatively minor for middle-aged workers who are beginning to accumulate retirement savings. In the last 5 years, however, vesting reduces participation more substantially, with declines of 4 percentage points for dropouts, 3.5 percentage points for high school graduates, and 2 percentage points for college graduates. Vesting has a larger effect in these periods because workers who lose their jobs later than age $64 - d$ will never vest before they reach retirement. Some of these individuals choose to save in the taxable account in order to replenish their precautionary savings against earnings fluctuations.

The coefficient of relative risk aversion σ affects both participation as well as the interaction between participation and vesting. Lowering σ increases the effect of vesting on participation for younger workers. When $\sigma = 1$, for example, participation decreases by 11 percentage points, 7 percentage points, and 16 percentage points for the three education groups when the vesting period rises from 0 to 4 years. This can be compared with the 4, 3, and 7 percentage point declines associated with the case of $\sigma = 2$ discussed above. The intuition for this result is that individuals who are less risk averse are willing to accept larger swings in consumption over time. When the expected return on 401(k) contributions is relatively low because of the risk of forfeiting matching contributions, these individuals would rather consume than contribute. Once vested, however, they compensate for their previous lower saving by reducing consumption in order to increase contributions. Another way to say this is that workers with lower risk aversion are more responsive to plan incentives such as vesting.

The main reason that workers choose to save in the taxable account instead of the 401(k) is liquidity. If withdrawals during employment are prohibited, then workers cannot use 401(k) savings to smooth consumption in response to shocks to earnings. But the assumption that withdrawals are prohibited during employment overstates the illiquidity of 401(k) plans. In reality, a growing number of 401(k) plans are offering plan loans, which allow individuals to borrow up to the lesser 50 percent of 401(k) savings and $50,000. We test how these loans affect our results by solving the model for a specification that allows unpenalized withdrawals during employment up to the maximum amount allowed by law.

Table 5 compares participation rates for an economy with and without loans for $\sigma = 3$.[29] The first thing to notice about the table is that loans *decrease* participation for many of the age and education groups. For example, the participation rate for high school graduates aged 40-49 falls from 100 percent to 81 percent. While this seems counterintuitive, it is easily explained as an artifact of our definition of participation as positive contributions. Plan loans effectively make the 401(k) a dominant vehicle for all types of saving motives—buffer stock as well as retirement. This means that some individuals who receive low earnings will now smooth consumption out of 401(k) loans instead of taxable saving. This tends to push down our measure of participation rates. But even with this effect, participation rises substantially for younger high school graduates and dramatically for younger college graduates. In the economy without loans, some of these younger workers saved only in the taxable account as a buffer stock against earnings volatility. With loans, these individuals now place buffer stock saving in the 401(k), boosting participation in the plan.

[29] We present results for $\sigma = 3$ because this was the original baseline parameter, and we did not solve the model for other combinations of the match rate, loans, and σ. Nevertheless, because higher values of σ make individuals less responsive to plan incentives, this choice implies that our results pertaining to the match rate and availability of plan loans can be treated as lower bounds.

How does employer matching affect participation and contributions? Table 6 displays participation rates by age and education for matching rates of 25 and 50 percent for $\sigma = 3$. For all ages, higher matching rates increase participation, but once again the effect is largest for younger workers. Participation among workers aged 20 to 29 increases by 13 percentage points for college graduates, 15 percentage points for high school graduates, and 8 percentage points for dropouts.[30] While matching appears to have a strong positive effect on participation, the results for contributions are ambiguous. Table 7 reports the average contributions for the two matching rates. Increasing the match rate pushes up contributions of younger workers—particularly college graduates—but it tends to lower the contributions of middle aged workers. This finding helps explain why some studies find that matching has only a weak effect on contributions.[31] The ambiguous effect of the match rate reflects the fact that the income and substitution effects of matching work in opposite directions.

Empirical studies examining 401(k) participation often point to low rates of participation among low-income workers as cause for concern that these workers will not accumulate sufficient wealth for retirement. But the results in this section suggest that lower rates of participation should be expected for liquidity-constrained younger workers. Munnell and Sundén (2004) report participation rates from the 2001 Survey of Consumer Finances (SCF) for different age and earnings groups. Table 8 reports their numbers. Comparing these numbers with the participation rates in Tables 4 though 6, it appears that our simulations overstate participation between ages 40 and 64 by about 20 percentage points. The model performs much better, however, for ages 20 to 39, especially considering that college graduates make up a disproportionate share of participants in the data.

5.4 Years of Limit Contributions and Participation

Another way to summarize the effects of matching, vesting, limits, and risk aversion on contributions and participation is to look at the average number of years people participate in the 401(k) and the average number of years they contribute at the matching and contribution limits. Table 9 displays these statistics by age and education groups for $\sigma = 1$, 2, and 3.

As in Table 4, vesting has a larger effect on participation for lower values of σ. For example, when $\sigma = 1$, increasing the vesting period from 1 to 3 years decreases college graduates' participation by almost 2 years. When $\sigma = 2$, the decrease falls to 1.5 years, and the effect is negligible when $\sigma = 3$.

Risk aversion also changes the incentive effects of matching and contribution limits. The

[30] Several empirical studies find that the presence of matching increases participation. The level of matching, however, appears much less important. See Munnell and Sundén, 2004.

[31] See, for example, Kusko, Poterba, and Wilcox (1998), Papke (1995), and Papka and Poterba (1995).

inverse of σ is the elasticity of intertemporal substitution, and individuals with lower values of σ are more willing to adjust their consumption levels in response to vesting and matching policies. That is exactly what we see in Table 9. For all education groups, individuals with lower values of σ end up contributing for more years near the matching limit and fewer years near the contribution limit. One way to characterize this strategy is that less risk-averse individuals smooth contributions instead of consumption and thereby take greater advantage of employer matching.

We now consider the effect of employer matching on the years of participation and limit contributions. Table 10 reports average years of participation and limit contributions for match rates of 25 and 50 percent and a coefficient of relative risk aversion of 3. Looking at the table, it is clear that the match rate has a strong effect on participation. Increasing the match rate from 25 to 50 percent increases the average number of years of participation by about 1 year for dropouts and around 2.25 years for both high school and college graduates. The higher match rate also boosts the number of years people contribute near the matching limit: 3 years for dropouts, 2.6 years for college graduates, and over 7 years for high school graduates. In contrast to the effect on matching limit contributions, the higher match rate actually decreases the number of years individuals contribute near the contribution limit. This makes sense. Individuals respond to more generous matching by spreading out their contributions over time in order to avoid contributing below the matching limit in some periods and beyond it in others. By evening out the contributions over time, this strategy has the added effect of making it less likely that an individual will be constrained by the contribution limit.

6 Conclusion

A commonly voiced concern about 401(k) plans is that participation among lower income workers is too low given the sizable benefits of contributing. Managers who must meet nondiscrimination rules set by ERISA, for example, are especially worried about boosting participation among low-earnings employees. Results from our simulation exercises indicate that lower participation for these individuals may be perfectly consistent with rational, utility maximizing behavior once one accounts for the shocks and constraints they face. In particular, the model in this paper predicts that younger workers will participate at a much lower rate than older workers. Participation for these workers is low for two reasons. First, they are liquidity constrained and save primarily as a precaution against earnings fluctuations, a role for which 401(k) plans are illsuited, given the withdrawal rules. And second, these workers have lower tenure and are consequently more likely to forfeit unvested employer matching contributions in the event of a job loss. Vesting rules reduce participation because the cost to workers of waiting to contribute until they are fully vested is reasonably small as long as they can make up lost employer matched contributions in the future.

The results from our simulations suggest that participation and contribution decisions should be sensitive to various plan features such as vesting, matching policies, and the availability of plan loans. Because these introduce distortions and kinks in the individual's saving decision and budget set, it is difficult to measure the elasticity of contributions to them using traditional econometric methods. In contrast, our life cycle model is able to account for the role of these nonlinearities in shaping contribution and participation behavior. Perhaps the most striking finding is the strong effect of the matching limit on contribution decisions. Conditional on participation, individuals contribute at the matching limit for 7 to 15 years, depending on education and income prospects. In addition, we also find substantial increases in participation for shorter vesting periods, larger match rates, and the availability of plan loans.

As with other representative agent models, our framework is subject to the criticism that we are really testing a joint hypothesis of rational behavior and a particular specification of constraints and parameters. We do not take a hard line on the rationality of savers—indeed, this assumption has been substantially weakened by Choi, Laibson, and Madrian's (2004) finding that seemingly irrelevant aspects of plan design, such as default policies, affect participation—but instead propose our model as a baseline against which future work can be measured. The other part of the criticism essentially addresses the question of what the model leaves out. Along these lines, one possible improvement may be adding a bequest motive to the model. Bequests would provide an additional incentive to save and presumably increase participation and contributions to the 401(k). But this is not clear since younger individuals who *receive* bequests would have a lower demand for precautionary saving. Another useful addition to the model would be introducing time-dependent utility, which would allow the model to account for important life-cycle events such as buying a home or paying for a child's college education. These events can play a key role in determining households' demand for liquidity, and therefore the cost of contributing to 401(k) plans, at different stages in the life cycle. Finally, a significant improvement would be modeling labor supply. Vesting reduces 401(k) participation in our model because of the risk of involuntary job separation. But the main reason firms adopt vesting rules is to reduce quitting. It would be interesting to examine the interaction between vesting and quitting in a model with outside wage offers. Modeling these additional features represents an important extension to this paper and is the next step in a research agenda aimed at understanding how plan features affect contribution and participation decisions.

References

[1] Abowd, J.M., and D. Card (1989), "On the Covariance Structure of Earnings and Hour Changes," Econometrica 57(2), pp. 411-445.

[2] Altonji, J.G. and L.M. Segal (1996), "Small Sample Bias in GMM Estimation of Covariance Structures," Journal of Business and Economic Statistics, 14, pp. 353-366.

[3] Bassett, W.F., M.J. Fleming, and A.P. Rodrigues (1998), "How Workers Use 401(k) Plans: The Participation, Contribution, and Withdrawal Decisions," National Tax Journal, 51(2), pp. 263-289.

[4] Bernheim, D.B. (1998), "Financial Illiteracy, Education, and Retirement Saving," in: O.S. Mitchell and S.J. Schieber, eds., *Living with Defined Contribution Plans: Remaking Responsibility* (University of Pennsylvania Press, Philadelphia), pp. 38-68.

[5] Bertsekas, D.P. (2000), *Dynamic Programming and Optimal Control* (Athena Scientific, Belmont).

[6] Carroll, C.D. (1992), "Buffer Stock Saving: Some Macroeconomic Evidence," Brookings Papers on Economic Activity,1992:2, pp. 61-156.

[7] Carroll, C.D., and M.S. Kimball (2001), "Liquidity Constraints and Precautionary Saving," NBER Working Paper No. 8496.

[8] Chamberlain, G. (1984), "Panel Data," in Z. Grilliches and M. Intrilligator, eds., *Handbook of Econometrics* (Amsterdam, North-Holland), pp. 1247-1318.

[9] Clark, R.L., and S.J. Schieber (1998) "Factors Affecting Participation Rates and Contribution Levels in 401(k) Plans," in: O.S. Mitchell and S.J. Schieber, eds., *Living with Defined Contribution Plans: Remaking Responsibility* (University of Pennsylvania Press, Philadelphia), pp. 69-97.

[10] Choi, J.J., D.I. Laibson, and B.C. Madrian (2004), "Plan Design and 401(k) Savings Outcomes," National Tax Journal 57, pp. 275-298.

[11] Dickens, R. (2000), "The Evolution of Individual Male Earnings in Great Britain: 1975-95," The Economic Journal, 110 (January), pp. 27-49.

[12] Employee Benefit Research Institute (2005a), "History of 401(k) Plans," *Facts from EBRI: An Update* (February), Washington.

[13] ——— (2005b), "Employee Tenure: Stable Overall, but Male and Female Trends Differ," *Notes* (March), Washington.

[14] Engen, E.M., W.G. Gale, and J.K. Scholz (1994), Do Saving Incentives Work?" Brookings Papers on Economic Activity, 1994:1, pp. 85-180.

[15] Farber, H.S. (1997), The Changing Face of Job Loss in the United States, 1981-1995," Brookings Papers on Economic Activity, Microeconomics, pp. 55-128.

[16] French, E. (2005), "The Effects of Health, Wealth, and Wages on Labor Supply and Retirement Behavior," Review of Economic Studies, 72, pp. 395-427.

[17] Gourinchas, P.O. and J.A. Parker (2002), "Consumption Over the Life Cycle," Econometrica 70 (1), pp. 47-89.

[18] Hinz, R.P. and J.A. Turner (1998), "Pension Coverage Initiatives: Why Don't Workers Participate?" in: O.S. Mitchell and S.J. Schieber, eds., *Living with Defined Contribution Plans: Remaking Responsibility* (University of Pennsylvania Press, Philadelphia), pp. 17-37.

[19] Holden, S., and J. VanDerhei (2005), "401(k) Plan Asset Allocation, Account Balances, and Loan Activity in 2004, EBRI Issue Brief No. 285.

[20] Hubbard, G.R., J. Skinner, and S.P. Zeldes (1995), "Precautionary Saving and Social Insurance," Journal of Political Economy 103(2), pp. 360-399.

[21] İmrohoroğlu, A., S. İmrohoroğlu and D.H. Joines (1998), "The Effect of Tax-Favored Retirement Accounts on Capital Accumulation," American Economic Review 88(4), pp. 749-768.

[22] Investment Company Institute (2000), "401(k) Plan Participants: Characteristics, Contributions, and Account Activity," ICI Research Series.

[23] Kusko, A.L., J.M. Poterba, and D.W. Wilcox (1998), "Employee Decisions with Respect to 401(k) plans: Evidence from Individual-Level Data," in: O.S. Mitchell and S.J. Schieber, eds., *Living with Defined Contribution Plans: Remaking Responsibility* (University of Pennsylvania Press, Philadelphia), pp. 98-112.

[24] Laibson, D.I., A. Repetto, and J. Tobacman (1998), "Self-Control and Saving for Retirement," Brookings Papers on Economic Activity, 1998:1, pp. 91-172.

[25] Lillard, L.A., and R. Willis (1978), "Dynamic Aspects of Earnings Mobility," Econometrica 46, pp. 985-1012.

[26] Love, D.A. (2006), "Buffer Stock Saving in Retirement Accounts," Journal of Monetary Economics, forthcoming.

[27] MaCurdy, T. (1982), "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," Journal of Econometrics, 18, pp. 83-114.

[28] Mitchell, O.S. (1999), "New Trends in Pension Benefit and Retirement Provisions," NBER Working Paper No. 7381.

[29] Munnell, A.H., and A. Sundén (2004), *Coming Up Short: The Challenge of 401(k) Plans* (Brookings Institution Press, Washington, D.C.).

[30] Munnell, A.H., A. Sundén, and C. Taylor (2002), "What Determines 401(k) Participation and Contributions?" Social Security Bulletin 64.

[31] Papke, L. E..(1995), "Participation in and Contributions to 401(k) Plans: Evidence from Plan Data," Journal of Human Resources. 30(2), pp. 311-25.

[32] Papke, L.E., and J.M. Poterba (1995), Survey Evidence on Employer Match Rates and Employee Saving Behavior in 401(k) Plans," Economic Letters, 49, pp. 313-17.

[33] Press, W., Flannery, B., Teukolsky, S., and Vetterling, W. (1992), *Numerical Recipes in C: The Art of Scientific Computing*, second edition. Cambridge University Press.

[34] Tauchen, G. and R. Hussey (1991), "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," Econometrica 59, pp. 371-396.

7 Appendix

7.1 Solution

There is no analytical solution to the dynamic programming (DP) model in this paper, so we must approximate a solution using numerical methods. The basic DP algorithm is described in Bertsekas (2000). Its application to our model involves first solving for the last period's problem at age 85, which produces a pair of optimal decision rules $c^*_T(\Omega_T)$ and $x^*_T(\Omega_T)$, and a corresponding value function $V_T(\Omega_T)$, where $\Omega_t = \{a_t, s_t\}_{t=65}^{t=T}$ is the discretized state space containing $N^a \times N^s$ elements. With these optimal decisions in hand, it is then possible to proceed backward in time to the retirement age, 65. At this point, the state space expands to accommodate several features that are particular to the working life, namely, earnings volatility, employment risk, employer vesting, and tenure. The expanded state space, $\Omega_t = \{a_t, s_t, m_t, \xi_t, emp_t, ten_t\}_{t=20}^{t=64}$, contains $N^a \times N^s \times N^m \times N^\xi \times N^{emp} \times N^{ten}$ elements. In practice, we set $N^a = 30$, $N^s = 20$, $N^m = 10$, $N^\xi = 2$, $N^{emp} = 2$, and $N^{ten} = 1 - 5$, depending on the vesting duration. For each member of the state space, we continue to solve the model backward, collecting separate decision rules and value functions for ages 64 to 20. Since there are 21 retirement years and 46 working years, the size of the state space requires that we solve our model for between $1,116,600$ and $5,532,600$ discrete points. The code, written in C++, is available from the author upon request.

Solving these types of DP models entails making important choices about numerical techniques, particularly with respect to optimization and interpolation. The optimization procedure we choose for the consumption and contribution decisions combines inverse parabolic interpolation and golden section search. For a given value of x_t, the value function appears to be nicely concave with respect to c_t. We can therefore locate optimal consumption using the relatively efficient inverse parabolic interpolation scheme, based on Brent's method, described in *Numerical Recipes for C.* Optimizing over contributions, however, requires a more robust method. For a given level of consumption, the tradeoff for increasing contributions is a reduction in conventional saving. Since these are often reasonably close substitutes, the value function is quite flat with respect to changes in x_t, making it difficult to locate the optimal level of contributions. Golden section search, which essentially translates bisection, a root-finding technique, to optimization, provides a robust but somewhat sluggish method for obtaining the optimal amount of contributions. We combine the two optimization techniques by defining a new function $V(x_t, \Omega_t)$ which returns a value function optimized over c_t. Golden section search can then be used to obtain the value of x_t that optimizes $V(x_t, \Omega_t)$.

Interpolation and extrapolation play a key role in the solution to our problem. At each age t, we optimize a value function that is exact only at the discretized points in the state space. Between and beyond these points, the value function must be approximated using interpolation and extrapolation. For working periods, interpolation applies to a cubic set of $N^a \times N^s \times N^m$ discrete points on the value function ($N^a \times N^s$ for retirement periods). We use cubic splines for assets in the 401(k) plan and the conventional account, and linear interpolation for the unvested portion of matched contributions. At the boundary points of this cubic set, we make the following two assumptions for extrapolation. First, the second derivatives at the lower bounds of the value function grid are zero. And second, the second derivatives of the value function at the upper bounds are equal to the second derivatives of the points just below the bounds. For the linear interpolation, this requires taking a second order Taylor approximation of the function at the upper bound of the m_t grid, where the second derivatives used in the approximation are themselves an approximation based on the change in the two previous first derivatives. Without proper handling of these extrapolated values, it is easy to overstate the value of increasing asset levels near the boundary points (e.g., a second derivative of zero implies no change in curvature). Since any errors in extrapolation cumulate over each successive period of the life cycle, these mistakes can cause first order problems in the solution.

Simulating the model once it is solved is relatively straightforward. The decision rules for consumption and contributions allow us to simulate, using Monte Carlo methods, the life cycles of people who differ in longevity, earnings, and employment histories. For each parameterization of the model we generate 20,000 such lives, which we use to derive the statistics reported in the main text.

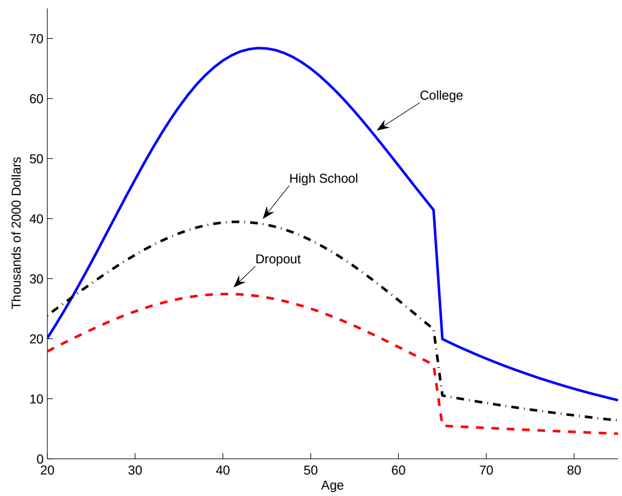

Figure 1: Earnings Profiles

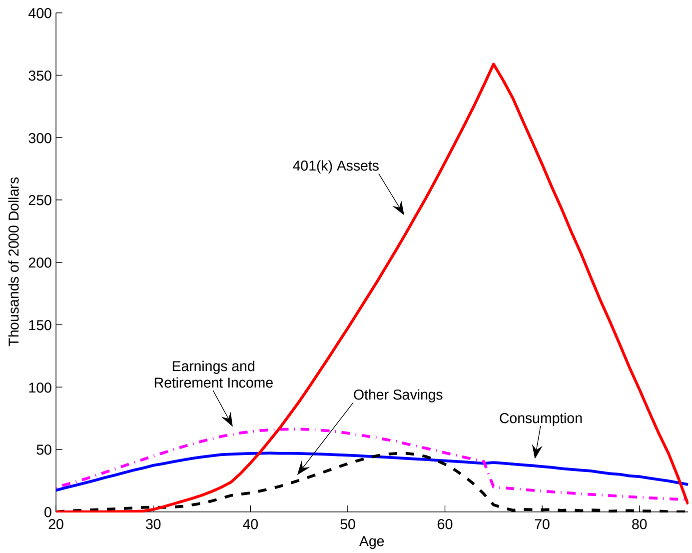

Figure 2: Average Life Cycle Profile of College Graduates

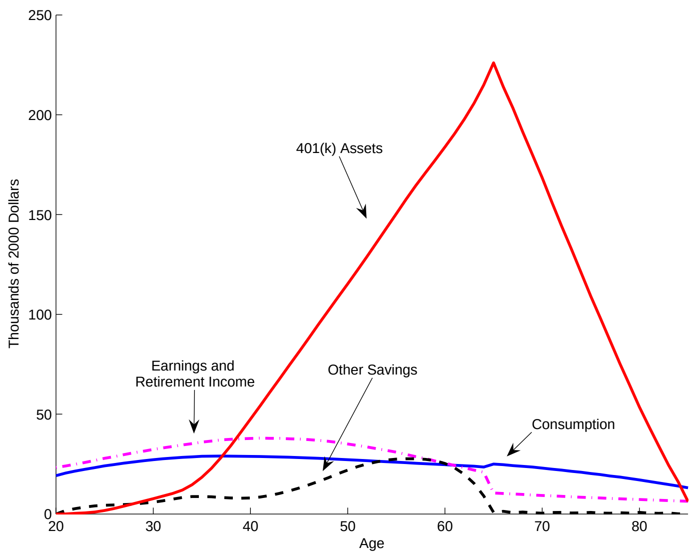

Figure 3: Average Life Cycle Profile of High School Graduates

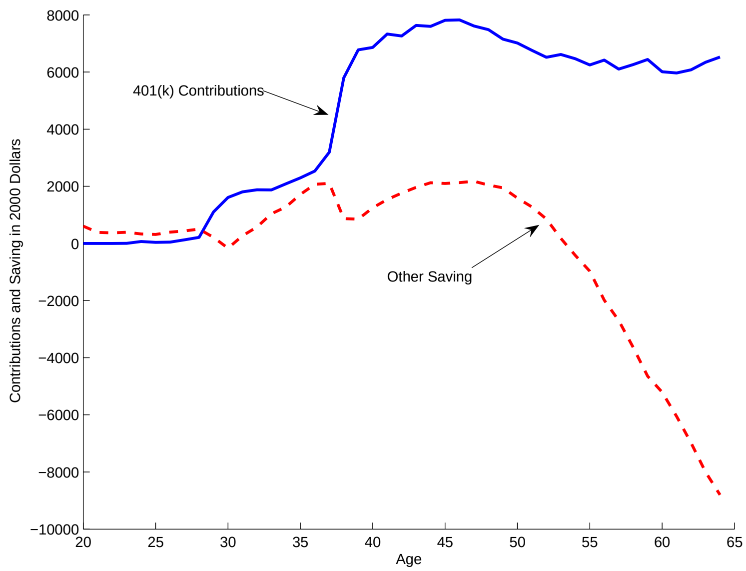

Figure 4: Contributions and Other Saving of College Graduates

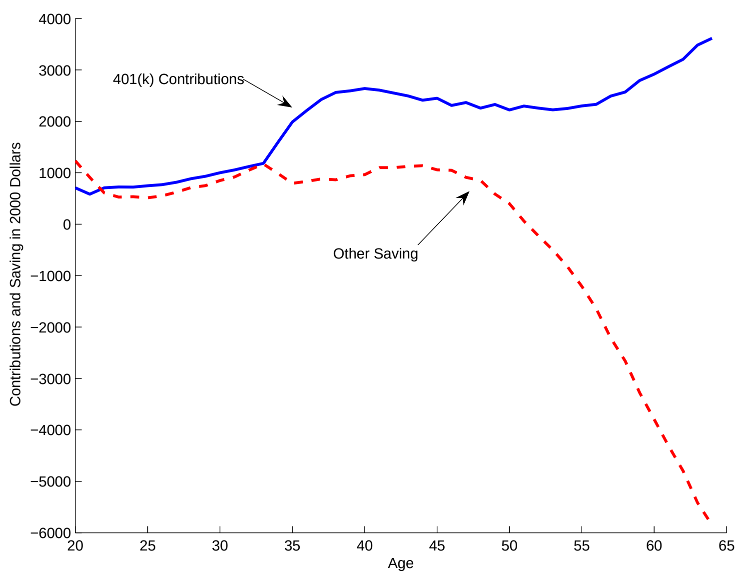

Figure 5: Contributions and Other Saving of Dropouts

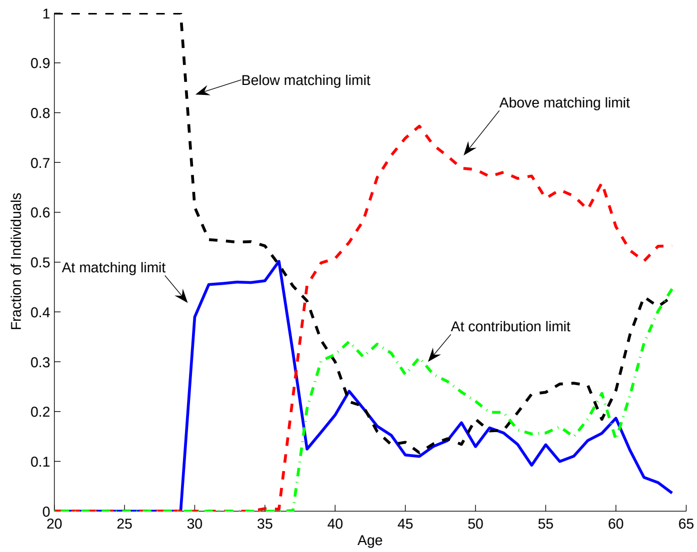

Figure 6: Fraction of College Graduates Contributing Near Limits

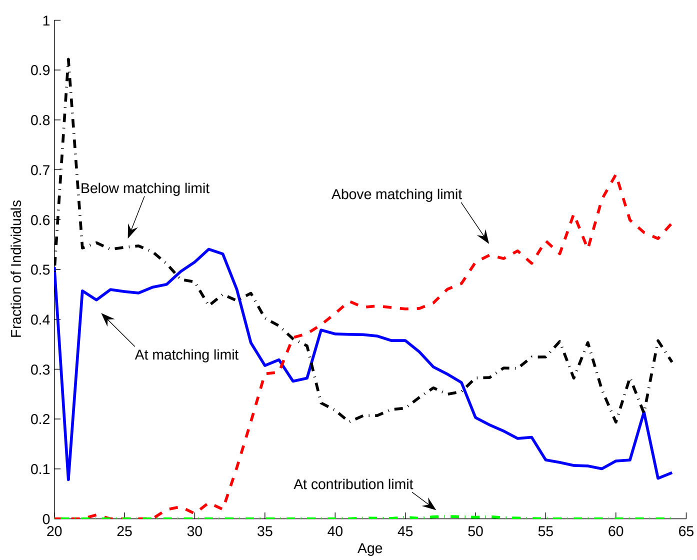

Figure 7: Fraction of Dropouts Contributing Near Limits

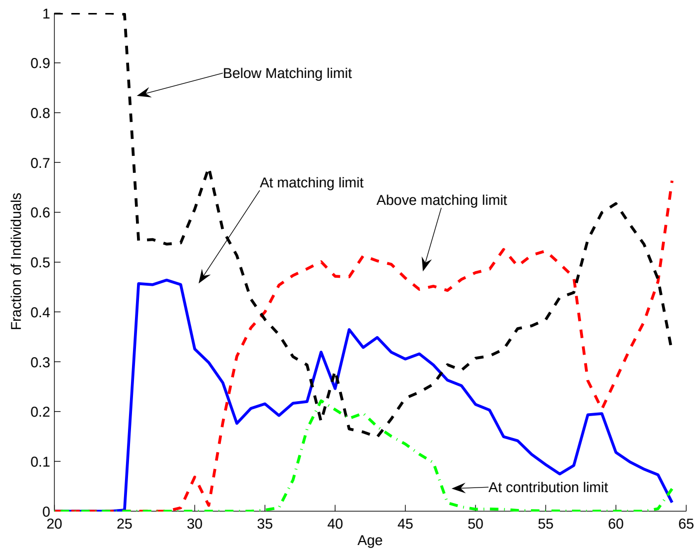

Figure 8: Fraction of High School Graduates Contributing Near Limits

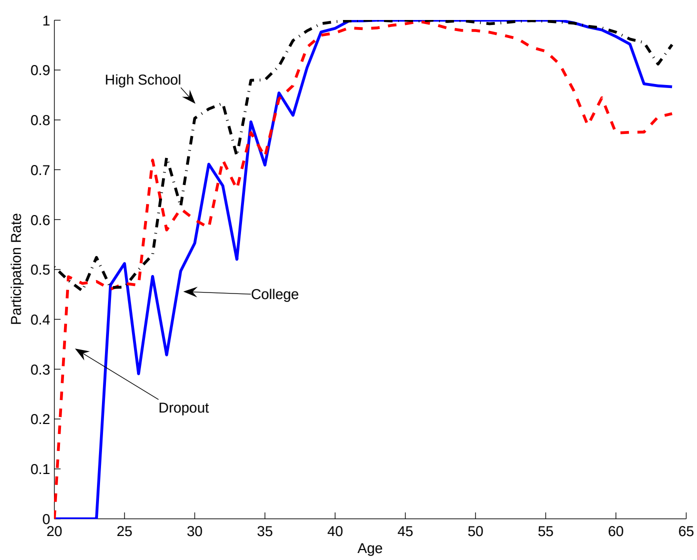

Figure 9: Participation Rates by Age and Education

Table 1: Cost of Waiting to Contribute $100 Until Vesting

t_v	Difference at t_v		Difference at $t_r = 20$		PV Difference		Percent Loss	
	$r = 3\%$	$r = 5\%$	$r = 3\%$	$r = 5\%$	$r = 3\%$	$r = 5\%$	$r = 3\%$	$r = 5\%$
1	$1.20	$2.00	$2.10	$5.05	$1.16	$1.90	0.58	0.96
2	$2.46	$4.18	$4.19	$10.05	$2.32	$3.79	1.18	1.93
3	$3.79	$6.55	$6.27	$15.01	$3.47	$5.66	1.77	2.91
4	$5.19	$9.12	$8.34	$19.92	$4.62	$7.51	2.36	3.90
5	$6.67	$11.92	$10.39	$24.79	$5.75	$9.34	2.96	4.90
6	$8.22	$14.95	$12.44	$29.61	$6.89	$11.16	3.57	5.90

For the simple example in the text, this table displays the costs associated with waiting to contribute $100 until reaching the vesting period t_v when the match rate is 100 percent. The first column shows the costs at time of vesting; the second column shows the costs at the point of withdrawal; the third column shows the present value of the costs; and the last column shows the costs as a percent:
$100\{(1+r)^{t_r})/([1+r(1-\tau)]^{t_v}(1+r)^{(t_r-t_v)})-1\}$.

Table 2: Coefficient Estimates for Earnings Profiles

Independent variable	Edu $<$ 12	Edu 12-15	Edu $>$ 15
Working individuals			
Age	0.0840	0.0778	0.2525
	(0.0266)	(0.0126)	(0.0256)
$\text{Age}^2/100$	-0.1057	-0.0727	-0.4064
	(0.0629)	(0.0305)	(0.0590)
$\text{Age}^3/10{,}000$	0.0030	-0.0329	0.1824
	(0.0468)	(0.0236)	(0.0438)
Race	-0.1222	-0.0688	-0.0400
	(0.0103)	(0.0050)	(0.0085)
Household size	0.0396	0.0344	0.0647
	(0.0055)	(0.0028)	(0.0049)
Constant, time, and cohort	8.4990	8.8132	6.3054
Retired individuals			
Age	-0.0138	-0.0251	-0.0357
	(0.0068)	(0.0071)	(0.0103)
Race	-0.0863	-0.0196	-0.2829
	(0.0322)	(0.0327)	(0.0744)
Household size	0.0771	0.0094	-0.0779
	(0.0234)	(0.0372)	(0.1022)
Constant, time, and cohort	9.5099	10.8936	12.2250
Error structure			
AR(1) correlation, ρ	0.8709	0.7803	0.8770
	(0.0254)	(0.0160)	(0.0136)
Transitory variance, σ_v^2	0.0844	0.0439	0.0326
	(0.0093)	(0.0027)	(0.0035)
Persistent variance, σ_ξ^2	0.0784	0.0571	0.0642
	(0.0097)	(0.0027)	(0.0040)

Source: Author's estimates using the 1980-1997 PSID. The top portion of the table displays the coefficient estimates and standard errors for the earnings regressions in Section 4.1, where the dependent variable is log earnings in year-2000 dollars for working individuals and log retirement income (Social Security plus defined benefit pensions) for retired individuals. The bottom of the table displays the GMM estimates of the covariance structure of earnings.

Table 3: Displacement Probabilities by Age and Education

Age	Dropout	High School	College
20-24	0.076	0.055	0.039
25-34	0.068	0.052	0.035
35-44	0.058	0.043	0.030
45-54	0.053	0.039	0.028
55-64	0.057	0.039	0.027

Source: Author's calculation using results from Tables A-3 and A-4 in Farber (1997). This table displays the probabilities of involuntary job separation for indviduals of different age and education groups.

Table 4: Percent Participating by Vesting Period

Age	Vest (Years)	Dropout			High School			College		
		$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
20-29	0	78.7	55.9	72.5	65.0	47.8	62.8	41.4	23.5	16.3
	2	73.3	52.8	71.6	62.8	44.5	60.6	33.5	19.2	13.7
	4	67.4	51.4	68.8	57.9	43.8	59.9	25.8	16.8	11.6
30-39	0	94.5	87.8	95.0	79.2	77.9	94.8	68.1	75.4	68.0
	2	92.9	87.3	94.4	78.1	75.7	93.2	66.6	75.0	67.6
	4	92.4	86.4	92.4	77.9	73.7	92.6	65.2	73.8	67.7
40-49	0	99.8	99.8	99.8	92.0	99.6	99.6	97.1	99.8	100.0
	2	99.5	99.7	99.7	91.2	99.1	99.8	96.8	99.2	100.0
	4	99.0	99.2	99.7	90.8	99.2	99.7	95.9	98.7	100.0
50-59	0	99.2	99.4	99.6	94.3	94.6	95.0	99.0	99.6	100.0
	2	99.5	99.6	99.5	93.2	94.2	95.1	99.6	99.9	100.0
	4	99.1	99.5	99.1	93.2	94.3	94.8	99.5	99.7	100.0
60-64	0	96.8	96.4	96.8	87.3	84.1	86.9	92.7	94.2	94.4
	2	95.0	94.9	96.4	86.7	82.9	86.4	92.4	93.5	93. 4
	4	93.6	92.1	93.6	84.9	80.5	85.1	91.0	92.4	92.8

Source: Author's simulations. This table displays the average percent of individuals of different age and education groups who participated in the plan, where participation is defined as positive contributions. Participation numbers are reported for vesting periods of 0, 2, and 4 and coefficients of relative risk aversion σ equal to 1, 2, and 3. The model was solved for a match rate $\mu = 0.25$.

Table 5: Percent Participating by Early Withdrawal Penalty

Age	Penalty Rate (Employed)	Dropout	High School	College
20-29	0	71	72	46
	1	74	63	16
30-39	0	76	69	66
	1	95	95	68
40-49	0	83	81	96
	1	100	100	100
50-59	0	65	61	87
	1	100	95	100
60-64	0	65	87	69
	1	94	84	94

Source: Author's simulations. This table displays the average percent of individuals of different age and education groups who participated in the plan, where participation is defined as positive contributions. Participation numbers are reported for two different assumptions about the early withdrawal penalty: early withdrawals made during employment are either costless ($\kappa = 0$) or prohibited ($\kappa = 1$). The model was solved with the following parameters: $\sigma = 3$, $\mu = 0.25$, and $d = 0$.

Table 6: Percent Participating by Match Rate

Age	Match Rate	Dropout	High School	College
20-29	0.25	74	63	16
	0.50	82	78	29
30-39	0.25	95	95	68
	0.50	97	96	72
40-49	0.25	100	100	100
	0.50	100	100	100
50-59	0.25	100	95	100
	0.50	100	99	100
60-64	0.25	94	84	94
	0.50	99	92	96

Source: Author's simulations. For a coefficient of relative risk aversion σ of 3 and immediate vesting, this table displays the average percent of individuals of different age and education groups who participated in the plan, where participation is defined as positive contributions. Participation numbers are reported for employer match rates μ of 0.25 and 0.50.

Table 7: Average Contributions by Match Rate

Age	Match Rate	Dropout	High School	College
20-29	0.25	819	807	226
	0.50	827	1,031	469
30-39	0.25	2,414	4,573	3,910
	0.50	1,903	3,512	3,387
40-49	0.25	2,602	4,302	8,241
	0.50	2,271	3,669	7,248
50-59	0.25	2,618	2,729	7,204
	0.50	2,578	2,946	6,494
60-64	0.25	3,510	3,134	6,518
	0.50	3,564	3,267	6,714

Source: Author's simulations. For a coefficient of relative risk aversion σ of 3 and immediate vesting, this table displays the average contributions of employed individuals of different age and education groups. Contribution numbers are reported for employer match rates μ of 0.25 and 0.50.

Table 8: Participation of Eligible Workers in 401(k) Plans

Age	Earnings in Thousands of Dollars					
	<20	20-39.9	40-59.9	60-79.9	80-99.9	> 100
20-29	36.9	65.9	72.9	86.7	*	*
30-39	49.9	69.8	78.5	87.0	81.8	91.4
40-49	55.0	73.2	80.7	80.0	92.6	96.2
50-59	68.4	70.6	73.5	72.7	95.6	87.2
60-64	*	73.2	76.5	*	*	*

Source: Munnell and Sunden (2004), p. 57, Table 3-2. This table shows the percent of eligible workers of different age and income groups who participated in a plan in 2001. The numbers are based on the authors' calculations from the 2001 Survey of Consumer Finances.

Table 9: Years Participating and Years Contributing at Limits

CRRA	Education	Vesting Period	Years Participating	Years at Match Limit	Years at Cont. Limit
$\sigma = 1$	Dropout	1	36.49	15.07	0.00
		3	35.52	14.26	0.01
	High School	1	34.37	12.18	0.20
		3	33.55	11.58	0.21
	College	1	32.55	9.55	1.28
		3	30.76	9.53	1.33
$\sigma = 2$	Dropout	1	36.00	12.00	0.01
		3	35.95	11.70	0.02
	High School	1	33.21	9.75	1.54
		3	32.95	8.75	1.72
	College	1	32.55	7.00	6.68
		3	31.02	6.86	5.52
$\sigma = 3$	Dropout	1	38.35	8.91	0.08
		3	38.25	8.16	0.10
	High School	1	36.93	7.27	2.15
		3	36.82	8.11	2.11
	College	1	31.55	6.20	9.66
		3	31.45	5.91	8.28

Source: Author's simulations. For coefficients of relative risk aversion σ of 1, 2, and 3, vesting periods of 1 and 3, and a match rate μ of 0.25, this table displays three statistics for individuals of each education group: (1) the average number of years the individuals participated in the plan, (2) the average number of years they contributed within 5 percent of the matching limit, and (3) the average number of years they contributed within 5 percent of the elective contribution limit.

Table 10: Years Participating and Years Contributing at Limits by Match Rate

Match Rate	Education	Vesting Period	Years Participating	Years at Match Limit	Years at Cont. Limit
$\mu = 0.25$	Dropout	1	38.35	8.91	0.08
		3	38.25	8.16	0.10
	High School	1	36.93	7.27	2.15
		3	36.82	8.11	2.11
	College	1	31.55	6.20	9.66
		3	31.45	5.91	8.28
$\mu = 0.50$	Dropout	1	39.42	12.10	0.02
		3	38.97	12.56	0.02
	High School	1	39.29	14.73	1.01
		3	38.96	14.45	0.89
	College	1	33.75	8.85	6.31
		3	33.69	8.64	5.06

Source: Author's simulations. For a coefficient of relative risk aversion σ of 3 and employer matching rates μ of 0.25 and 0.50, this table displays three statistics for individuals of each education group: (1) the average number of years the individuals participated in the plan, (2) the average number of years they contributed within 5 percent of the matching limit, and (3) the average number of years they contributed within 5 percent of the elective contribution limit.